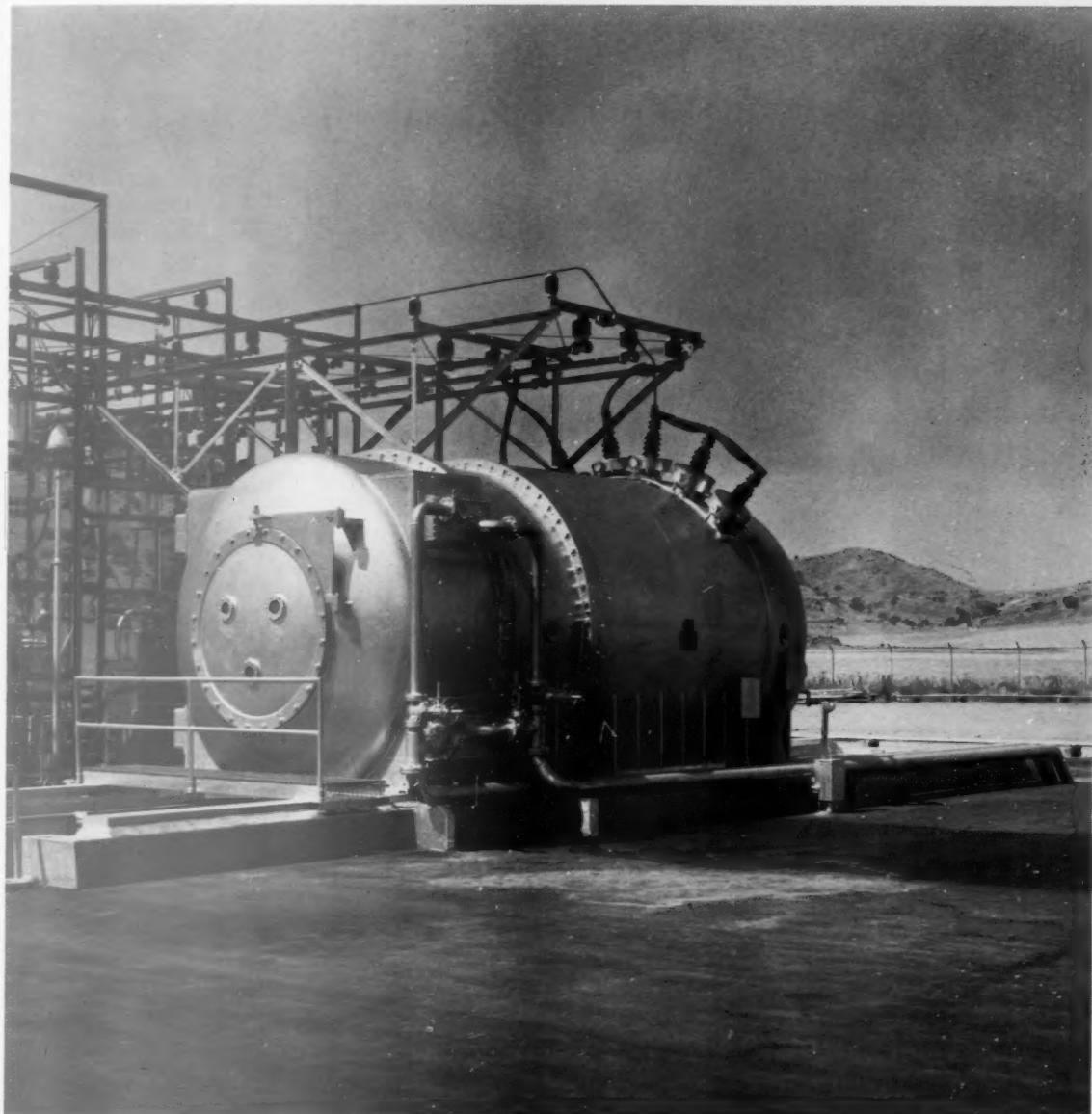


ALLIS-CHALMERS

# Electrical Review



*Third Quarter, 1950*

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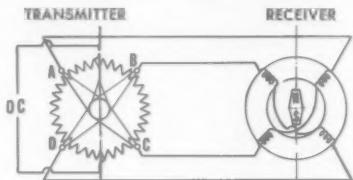


FIG. 1

D.C. voltage is applied to the transmitter resistance bridge at fixed points 180° apart. Roller brushes A, B, C, and D are mechanically positioned 90° from each other and can be rotated on the resistance bridge to change their position relative to the D.C. taps. These roller brushes are connected to receiver windings as shown in diagram 1.

The receiver rotor, being permanent magnet, aligns itself with the center-line of the resultant stator poles. These resultant poles depend on the amount and direction of current flowing through the receiver stator coils. The current in turn depends upon the relative distance the transmitter rotating taps are from the fixed D.C. taps.

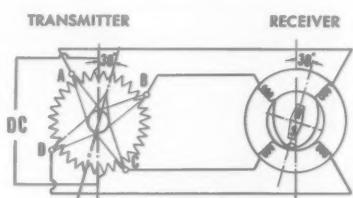


FIG. 2

Rotating transmitter taps A, B, C, and D 30° clockwise (fig. 2) rotates the receiver resultant poles 30° and causes a 30° clockwise rotation of the permanent magnet armature. In a like manner, any degree of rotation of the transmitter taps will be accurately indicated by the receiver.



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Allis-Chalmers

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# ALLIS-CHALMERS Electrical REVIEW



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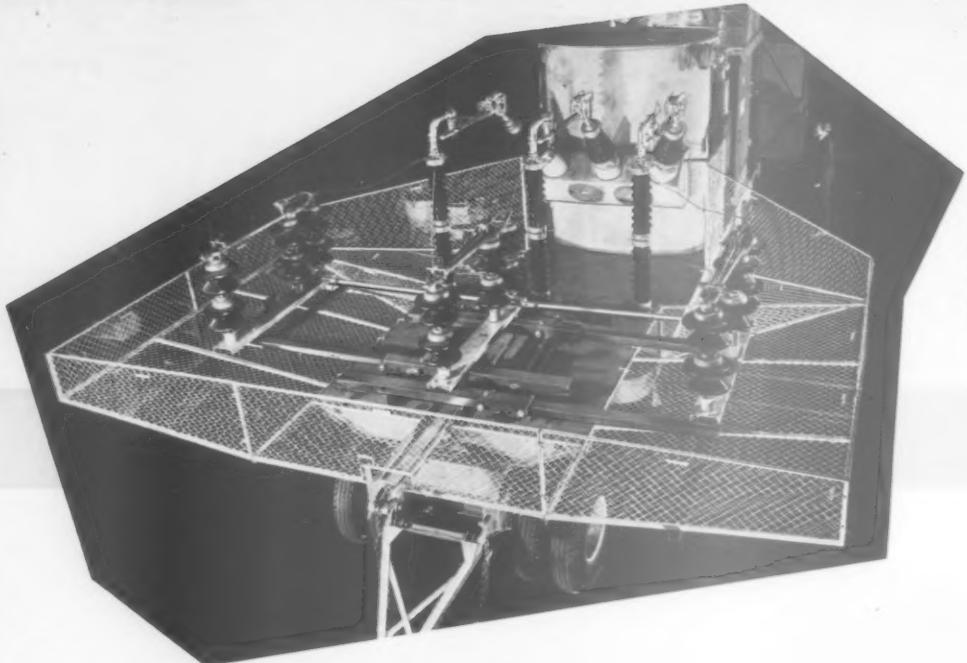
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**THE ELECTRICAL REVIEW** will be available to libraries on microfilm as a result of an arrangement between Allis-Chalmers Mfg. Co. and University Microfilms of Ann Arbor, Michigan. To solve the critically acute shortage of storage space, libraries and other institutions are microfilming periodicals, thus utilizing only about one-twentieth the space required for storing originals. All issues of the **ELECTRICAL REVIEW** will be microfilmed at the end of each year and positive films will be available, to anyone normally receiving the publication, at nominal cost. Inquiries about service should be directed to University Microfilms, 313 N. First St., Ann Arbor, Michigan.



**READY FOR OPERATION,** this 66-kv mobile unit substation needs only incoming high voltage and outgoing low voltage lines to meet any power emergency. Protective fence around the trailer-mounted switchyard protects personnel against contact with live parts. (FIGURE 1)

# Take the Switchyard Along

by **FRANK J. ARNAUD**

Transformer Section  
and

**ROBERT LOEWE**  
Switchgear Section  
Allis-Chalmers Mfg. Co.



*Taking a lesson from the Pullman berth, substation designers now provide complete, fold-up switching facilities for modern mobile unit substations*

**M**OBILE UNIT SUBSTATIONS are meant to be moved occasionally wherever and whenever emergency and temporary use warrant. This feature used at least a reasonable number of times during the lifetime of the equipment justifies the increased cost for mobility.

Occasional moving of the substation from one place to another implies that each particular application is only temporary. Replacing an existing substation that is being repaired, enlarged or modified is one of the most obvious uses of mobile unit substations, although it is not the most frequent. In the past these have been used chiefly to provide temporary service during the procurement, erection and connection of permanent equipment, or to provide temporary service at resort areas, lumber camps, construction projects, etc.

In replacing an existing substation, it would be too costly to duplicate the high voltage switching and protective equipment usually provided in a substation since this equipment is not particularly subject to permanent damage or to obsolescence by the growth of the load. Also, temporary damage to such equipment can be repaired quickly and easily. Any inadequacy due to load growth can be remedied easily by the substitution of new components for some of the elements which, while major in their functions, are minor in size, cost, and difficulty of replacement.

If mobile unit substations were limited in use to provide temporary replacement in an existing substation, it would then be wise to use the high voltage switching and protective equipment already installed and connected at the location where such service is needed. Under these conditions, the mobile unit substation need include only the transformer and low voltage switchgear since, from the standpoint of difficulty of repair and replacement, these are the most vulnerable elements of a substation. Power systems may have one or more locations where use of such existing equipment would be inconvenient but this would not cause any great hardship for operating personnel nor would it warrant the cost of duplicating unnecessary equipment.

## Primary switching equipment needs room

An entirely different situation arises when it becomes necessary to move a mobile unit substation into Farmer Jones' cornfield to serve a new load temporarily. The erection of a complete switching structure like that shown in Figure 2 would be too costly in both time and money unless it could be used with a permanent substation or the incoming voltage was so high that the mounting of the high voltage switchyard with the rest of the substation for portability was impractical. For maximum usefulness a substation of this sort must be a complete, integral unit capable of being moved to the desired location,



THIS 110-KV mobile unit substation operating at a dam site shows the permanent structure required when the high voltage switching equipment is not trailer-mounted. In this case, however, the switching equipment will be used with another permanent installation in the future. (FIGURE 2)



MOBILE UNIT SUBSTATIONS for low voltage distribution, like this 13.2-kv to 2.4- or 4.16-kv class, are as complete as the larger type. At these low voltages, clearances offer no unusual design problems. Parts are designed for full safety and immediate operation on a moment's notice.

connected to any convenient adjacent high voltage lines to transform and transmit power in usable form to the low voltage lines.

To comply with all of these requirements, a mobile unit substation for such applications must include a high voltage switchyard containing the necessary high voltage switching and protective equipment, transformer to reduce voltage to a more usable value preferably with several of the more frequently used secondary voltages, and low voltage switchgear for the control, metering, and protection of outgoing power. All equipment must be mounted on a vehicle designed for towing over roads of the area served by the utility. Conformance with laws governing vehicular traffic of the states over whose roads the substation must travel is essential. These laws specify overall length, weight, and, more restrictive to mobile unit substation design, the overall width and height. They also require that clearance, warning, and signal lights must be supplied according to legal specifications.

These dimensional restrictions do not pose a serious problem for the design engineer in the planning of lower voltage substations and those which do not include the high voltage switchyard. NEMA requirements for interphase and phase-to-ground spacing distances of bushings, switches, and other live parts for the higher voltage substations with complete switchyards, however, are incompatible with road clearance laws. Since these larger substations (Figure 1) consist of a greater variety of equipment, special consideration must be given to the design of components as well as their arrangement.

The 3,000-kva, three-phase, 60-cycle unit shown in Figure 1 is mounted on a trailer of suitable size and strength required for permanent mounting and safe legal transportation of the various components as a single unit over roads of the area in which the substation is to be used.

When operating as a substation, the individual pieces of equipment do not rest directly on the ground but are connected directly to the trailer and must be grounded through a ground wire which may be as far as 20 feet away from the most remote portions of the unit. It is possible, therefore, to get higher than usual earth gradients and voltages from the steel framework to earth at points remote from the grounds. Therefore, to provide greater protection for personnel, a complete high voltage unit must be supplied with unusually good grounding both in number and low resistance of grounds used. This extra protection is provided by three ground pads which are placed on the trailer frame, one on each side and at the beginning of the gooseneck and one in the center rear of the

trailer. In certain semi-permanent locations it might be desirable to have a buried ground wire connecting all three pads in order to reduce earth gradients.

The trailer is also equipped with the necessary clearance, warning, and signal lights and four jacks to take the weight off the tires.

### Special designs save space

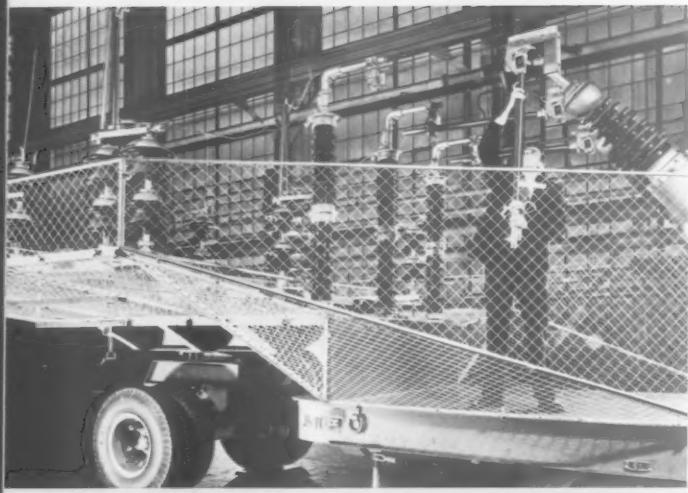
The air-break horn-gap type 69-kv high voltage disconnect switch is shown mounted on the front part of the trailer, Figure 1. NEMA standards for 69-kv switches of this type require an 84-inch phase-to-phase spacing so that the overall width of the entire switch is approximately 15 feet when in operating position. Obviously, it is impossible to move the unit from one location to another with a switch of this type permanently mounted.

The two outer phases of the switch are mounted on telescoping draw-out carriages to reduce the work required to put the substation in operation or to ready it for transportation. These can be pushed in to road clearance width for transit and drawn out to the required operating clearances. When telescoped for movement, the switch carriage must be securely locked to prevent vibration and breakage. When the switch carriages are extended, they must be accurately positioned and securely locked in place so that the operating linkages extending from the center phase to the two outer phases toggle properly at the same point in the switch throw and hold the switch closed against forces likely to develop under fault conditions.

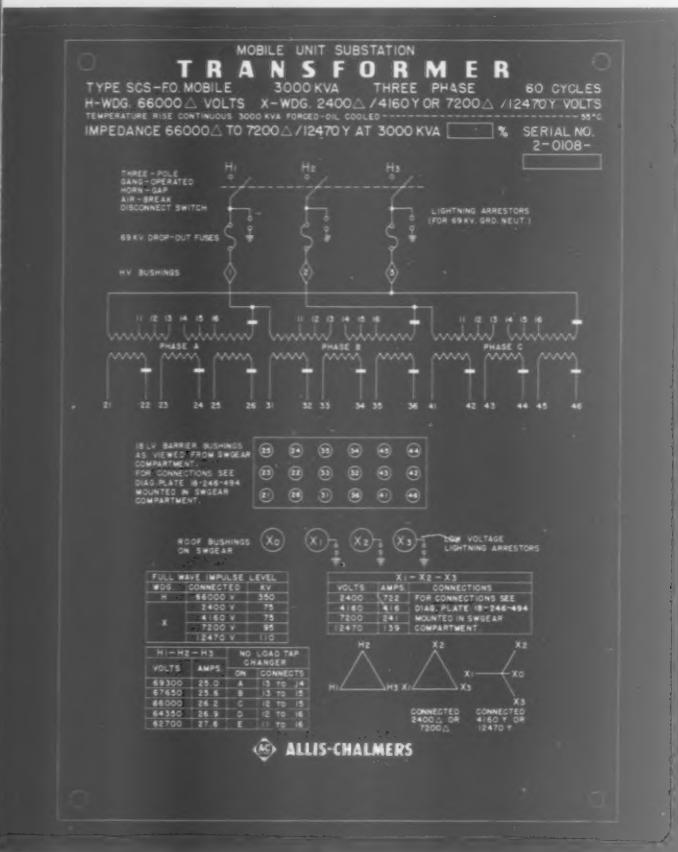
The mounting of the operating handle of the disconnect switch on a removable channel extending through the protective fence permits operation of the disconnect switch from outside the enclosure. The switch handle may also be locked in either position with a slip bolt and padlock.

The three lightning arrestors, connected next in the circuit from the incoming high voltage lines, are immediately behind the disconnect switch. Since the arrestors can be kept within the permissible eight-foot road clearances, they need not be removed for transportation. The arrestors are securely braced against longitudinal and transverse stresses by a set of easily installed and removed double triangular M-shaped braces between the tops of the lightning arrestors and stationary parts of the switch draw-out carriage.

Next in the circuit from the incoming high voltage lines to the transformer bushings are the high voltage fuses suspended between the lightning arrestors and bushings. These 69-kv



**FUSES CAN BE INSTALLED** or changed easily and safely. After opening the disconnect switch, the fuse may be removed safely by merely lowering to the vertical position and lifting the fuse out of the hinge clip. A standard hook stick can be used if desired. (FIGURE 3)



**ELECTRICAL INFORMATION** for the proper connection and operation of the mobile unit substation is contained on a stainless steel transformer nameplate. Two additional instruction plates are provided on the draw-out carriages to facilitate station set-up and dismantling. (FIGURE 4)

drop-out type fuses have an interrupting capacity of 500,000 kva. When a fuse link blows, the clip at the lightning arrestor end releases the fuse which drops out of the circuit and hangs by the hinge end on the transformer bushing. The fuse may be changed as shown in Figure 3 after the unit is de-energized by opening the incoming high voltage disconnect switch.

The transformer high voltage bushings are mounted in a bushing pocket low on the forward face of the transformer tanks. These are set at a height that brings the fuse hinge mounted on the bushings and the fuse clip on top of the lightning arrestors to the same level. All the bushings are tilted forward, with the two outer bushings tilted away from the center bushing to achieve the standard "desirable distance" specified by NEMA to be maintained from bushing to bushing, from bushings to the transformer tank wall, from the lower end of the fuse when in the discharged position to the transformer tank wall, and from the end of the fuse to the trailer platform.

### Protective enclosure twenty feet wide

All of the high voltage equipment, including the transformer high voltage bushings, is completely enclosed by a protective fence which conforms with NEMA's standard "desirable distance" recommendations between live parts and ground. Extending nine feet above ground level, it is impossible for even a tall man to reach over the top of the enclosure. When set up in operating condition, with the fence around it, the substation has an overall width in excess of 20 feet around the outside of the enclosure.

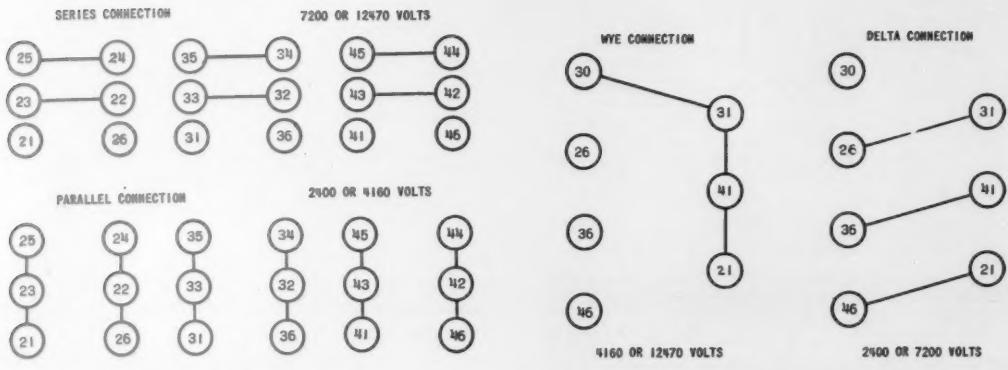
Built sectionally, the fence may be assembled or dismounted by two men. One man could handle the job in an emergency. When dismounted, the fence panels may be stowed for transportation in the well space forward of the transformer.

With the jacks raised and swiveled out of the way, the protective fence removed and stowed, disconnect switch telescoped, lightning arrestors braced, and the fuses removed, the remainder of the unit can be moved in the operating condition. The relays, however, must be blocked against vibration.

The transformer, heart of the substation, is a three-phase, 60-cycle, core type, forced-oil-cooled unit rated 3,000 kva. The high voltage winding is 66,000 volts delta with two 2½-percent full capacity taps above and below the nominal value. These are changed by a no-load high voltage tap adjustor.

The low voltage winding has four voltages; 2,400 or 7,200 volts delta or 4,160 or 12,470 volts wye, all full capacity. These voltages are obtained by winding the low voltage in three sections as shown on the nameplate diagram, Figure 4, so that by connecting all three windings of one phase in parallel 2,400 volts is obtained. By connecting all three in series 7,200 is obtained. All the low voltage winding leads (6 per phase, 18 in all) are brought out through the side of the tank by 18 barrier bushings in the throat between the transformer and switchgear. The throat is an air-filled chamber with a removable gasketed cover which permits ready access to make or change the series-parallel connections on the ends of the barrier bushings. Mounted within this space is a diagram plate, Figure 5, showing the proper connections between the bushings to obtain either series or parallel connection.

The inter-phase connections can be made in delta or wye in another terminal board to increase the number of available voltages to four. The bringing out of all 18 low voltage winding leads and the removal of all potential, current and



**SERIES PARALLEL** connections to change the phase voltages from 2,400 to 7,200 volts are made on the ends of the transformer barrier bushings as shown at left. Inter-phase connections (right) are made

on a terminal board shown in Figure 7. This provides 2,400 or 7,200 volts delta or 4,160 or 12,470 volts wye. Diagrams like these are provided to facilitate setting up the station for use. (FIGURE 5)

auxiliary power supply transformers from the main transformer tank to the switchgear units improves the substation's reliability considerably. Such placement of components greatly aids changing connections, inspection or repair. Because of this, tanks can be sealed permanently so that, when once purged with a dry inert gas, the heating and cooling of the oil with attendant expansion and contraction merely increases or decreases the pressure of the gas above the oil; eliminating inert gas pressure valves and other delicate devices.

The main transformer core and coils were designed particularly to reduce dimensions. This, and the fact that the only other item included in the main tank is the no-load tap adjustor, permits the use of a tank small enough to mount the transformer transverse on the trailer. This also reduces both length and complexity of the leads and increases reliability.

Heat generated in the transformer is dissipated by two Electro-coolers mounted one on each side of the transformer. Control for the cooler is mounted in the switchgear where it is conveniently adjacent to all other controls of the unit.

### Switchgear is compact but accessible

Figure 6, a single line diagram, shows the chief items of the single switchgear cubicle which is welded to the transformer, and contains all secondary control and metering. Each section of the secondary equipment is contained in its own individual and isolated section of the cubicle. Each is separated from other sections by full and complete metal barriers and is accessible through weather-proof hinged doors provided with easily operated handles which can be locked with any standard padlock. Ventilating louvers in the doors screen and filter the air so that dust and insects or other air-borne contamination does not enter the cubicle.

Braced and permanently insulated connections are brought through to the primary terminals of three double secondary current transformers from one of the bottom bushings of each phase of the series-parallel board. Three other connections

are brought similarly to the terminals of the wye-delta terminal board.

The current transformers are in the central compartment of the cubicle, which also contains other insulated bus connections.

Leads from the other terminals of the current transformers lead to the wye-delta terminal board shown in Figures 7 and 8. Figure 7 also shows a partial view of one of the Electro-coolers previously mentioned. This same compartment, but with a protective insulating barrier in front of the terminals, is shown in Figure 8.

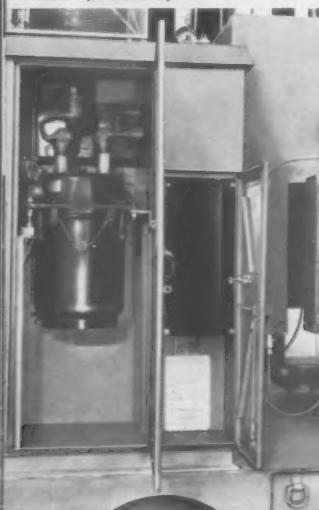
Figure 8 also shows the adjoining compartment which contains the tank and bushings of the oil circuit breaker. Rated 15 kv, 1,200 amp, the breaker has an interrupting capacity of 150,000 kva. The tank may be lowered and removed for inspection and maintenance of breaker parts by a crank operated, windlass type tank lifter mounted in the control compartment immediately to the left. The removable cover to the left of the door provides easy access to the auxiliary switches of the breaker for inspection and maintenance. This compartment also contains the solenoid operator for the breaker, tank lifter, breaker control relays, rectifier to provide direct current for breaker closing, and the capacitor tripping device which stores energy for opening the breaker.

Figure 8 shows the next section to the left which is isolated and contains two control power transformers and their primary fuses. These disconnect fuses are between insulating barriers and can be handled with a conventional hook stick. Rated 15 kv, 3 N amp, these will interrupt the highest short-circuit available within the station. The two control power transformers are located immediately below the fuses. Each transformer is rated 5 kva with series-parallel secondaries for 120/240 volts and three-part series-parallel primaries for either 2,400 or 7,200 volts.

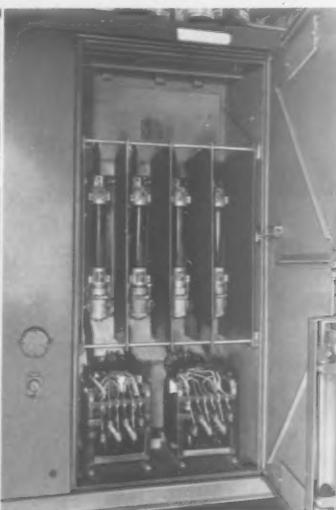
The main control section, shown in Figure 10 and to the left of the previous section, has a mounted panel hinged on



**WYE-DELTA BOARD** is shown exposed for changing. Warnings to operating personnel are provided for maximum safety during operation. (FIGURE 7)



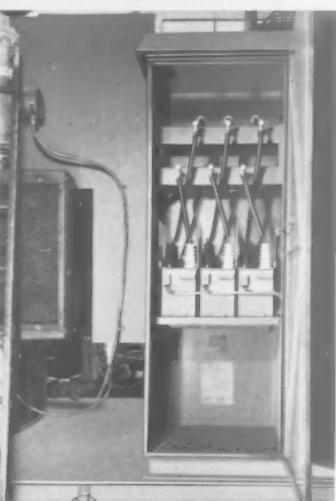
**BREAKER TANK**, bushings, and hv connections are easily accessible. Stop mechanism prevents doors from swinging when open. (FIGURE 8)



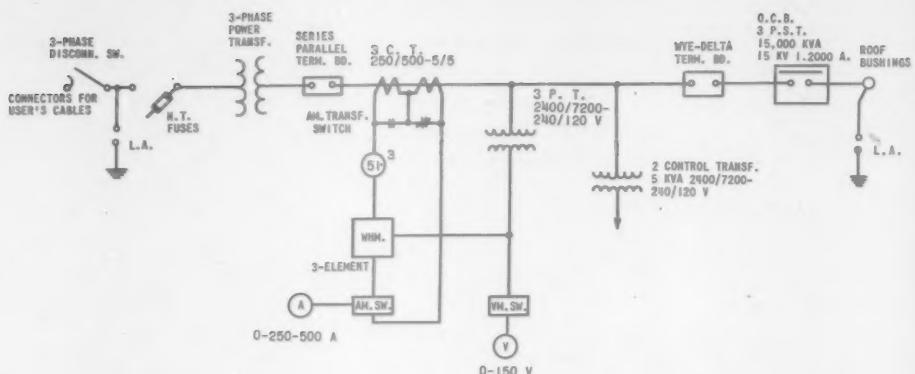
**STATION SERVICE** transformers are isolated. HV connections are insulated or protected from other live parts by insulating barriers. (FIG. 9)



**SECONDARY CONTROL** and metering are in one section. Equipment is arranged for easy accessibility, maximum efficiency, and safety. (FIG. 10)



**POTENTIAL TRANSFORMER** arrangement facilitates changes. Current limiting fuses are mounted for easy inspection and change. (FIG. 11)



**MAIN COMPARTMENTS** and their relations to each other are shown in the single line diagram above. For simplicity of installation and operation, efficient and

economical power distribution, and ease of transportation only the most essential items should be built into a mobile unit substation. (FIGURE 6)

the left for easier inspection and contains the ammeter, voltmeter, and associated switches for reading values in any phase. The panel also contains all necessary relays, meters, switches, controls, and indicating and alarm apparatus.

An alarm bell, breakers for the various control circuits, switches and starters for the Electro-cooler pumps and fans, current sensitive relays to indicate their normal operation, and various control circuit fuses are also mounted in this section. These, however, are not located on the panel.

Figure 11 shows the potential transformers and their current limiting primary fuses. Each transformer is provided with six primary bushings so that their three-part primaries may be used in either series or parallel connection, as shown on the plastic diagram. The removable metal access plate below the transformers provides entrance to the center section for maintenance and repair, and is sufficiently large to permit a man to work on all the high voltage bus work contained within it.

#### Components arranged for easy maintenance

Each section of the cubicle is equipped with an electric light to facilitate inspection, operation, and maintenance. Certain critical sections have electric strip heaters under manual control.

Four 15-kv, 1,200-amp bushings, including one for the neutral, with solderless connectors for the user's connections are mounted on the roof. Three lightning arrestors are connected to the bushings by a combination of flexible and solid copper. The arrestors are in two sections, stacked one upon the other, with a 2.5-kv section being placed upon a 5-kv section. A shorting connection is provided for the bottom or 5-kv section during use at 2,400 or 4,160 volts. Also on the roof, but at the rear edge, is a weather-proof enclosed fixture for yard lighting. All roof equipment is surrounded by a metal fence permanently mounted and of adequate height to provide protection against accidental contact with live parts.

The mobile unit substation is constructed so that all high voltage apparatus is effectively and safely isolated from the control sections. Complete warnings and instructions are provided for the different steps in installing the substation and placing it in service, disconnecting or changing voltage. Devices are provided for giving short circuit and overload protection to the different pieces of apparatus. The switchgear is built largely of standard parts enabling quick and easy replacement if damage should occur.

# Predetermining Voltage Distribution in **TRANSFORMERS**

by

**L. C. AICHER**  
Transformer Section  
Allis-Chalmers Mfg. Co.

*Versatile calculating board solves complex transformer problems in fraction of time required for ordinary calculations*

THE PHENOMENA occurring within transformer windings when lightning strikes establish the insulation requirements of the windings. Mathematical methods which aid in analyzing the distribution of the surge voltage within a transformer have been available for years. These, however, are usually too slow in providing the transformer design engineer with usable data for accurately determining the proper insulation for a particular unit.

Various simplified methods have been introduced through the years to ease the transformer design engineer's calculating problem. These, of course, have been developed around certain assumptions and have aided greatly in reducing the time required to reach a conclusion. Their main weakness is that, as transformers become more complicated, even these simplified methods are apt to be considerably burdensome. Also, some of the assumptions cease to apply as the designs become more complex.

These shortcomings have promoted the use of the calculating board. A miniature circuit that is the electrical equivalent of the many paths through which dielectric current flows under abnormal voltage conditions is set up on this board. A voltage is applied and measurements are made to show how a transient voltage is distributed through the windings and insulation of a transformer.



**CALCULATING BOARD** shows a typical transformer equivalent circuit connected for initial voltage measurements. A calibrated potentiometer (nearest author) and a milliammeter for equivalent circuit measurement are also shown. (FIG. 1)

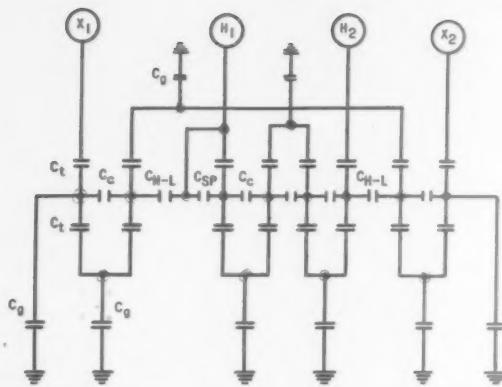
The calculating board illustrated in Figure 1 is not a gargantuan affair like those publicly acclaimed during the last several years. By comparison, it is a much smaller and considerably simpler device built to accomplish its special task.

## Board simulates any capacitance network

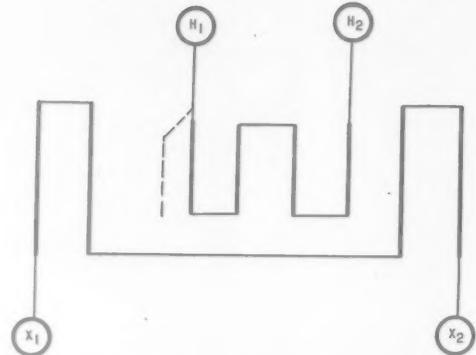
It can be shown that the maximum voltage gradient between any two points within a transformer winding is related to the distribution of voltage at the initial instant. Further, it is known that at this initial instant the voltage distributes through the transformer windings according to the behavior of a very intricate capacitance network. At this instant the inductive reactance of the windings is so great that it has little effect on the voltage, so it is omitted. This condition permits the use of a capacitance equivalent circuit similar to Figure 2a, representing the simple transformer winding shown in Figure 2b. If all transformer equivalent circuits were as simple as Figure 2a, there would be much less need for a calculating board.

Figure 3 is a typical example representing the actual capacitance network encountered in the analysis of a recently designed high voltage power transformer. Some are considerably more complicated than Figure 3. It is easy to appreciate why such a network becomes a burden when one wishes to know the voltage at a number of points within this circuit.

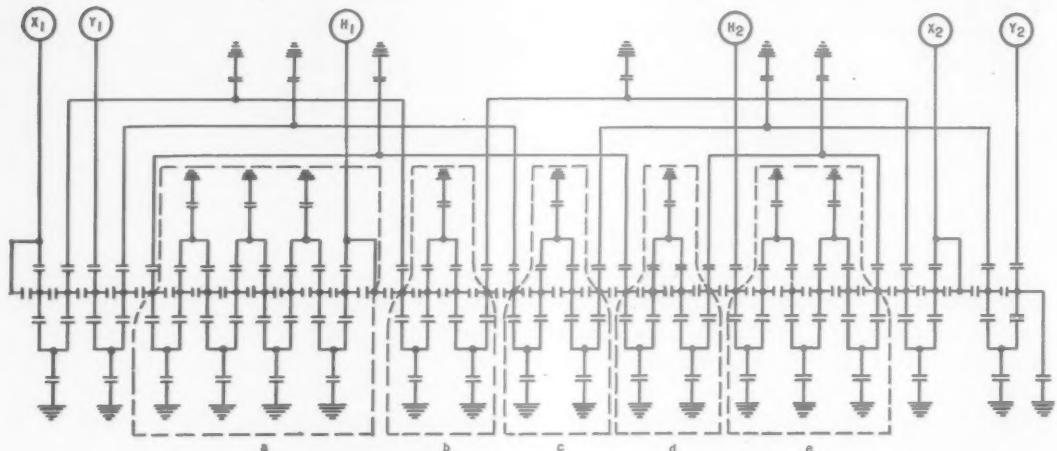
A calculating board could be built using a number of capacitors which could be connected into the various circuit arrangements. However, capacitors are relatively expensive elements. The calculating board shown in Figure 1 is a direct-current unit in which resistors are used to simulate capacitors.



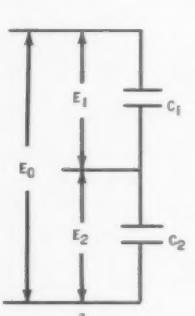
**EQUIVALENT CIRCUIT** of a two high-low group shell type transformer as used for initial voltage distribution measurement.



Simple equivalent circuit at left, representing the transformer winding at right, simplifies calculations. (FIGURES 2a and 2b)

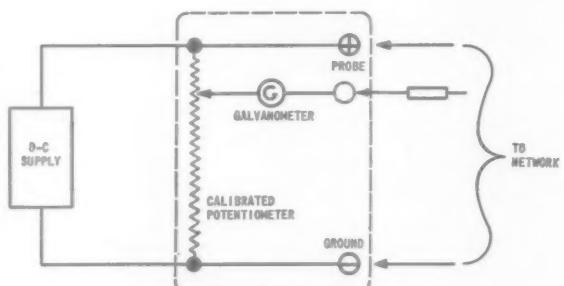


**MODERATELY COMPLEX** three-winding transformer with graded high voltage insulation complicates calculations. (FIGURE 3)

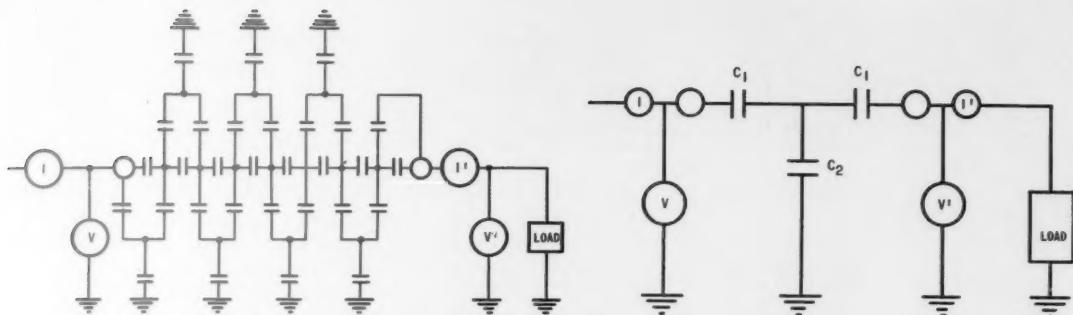


$$\begin{aligned}E_0 &= E_1 + E_2 \\E_0 &= \frac{1}{\omega C_1} + \frac{1}{\omega C_2} \\E_1 &= \frac{1}{\omega} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \\R_1 &\propto \frac{1}{C_1} \\R_2 &\propto \frac{1}{C_2}\end{aligned}$$

**RESISTIVE CIRCUIT** shown in (b) divides the voltage similarly to the capacitance circuit (a). (FIGURES 4a and 4b)

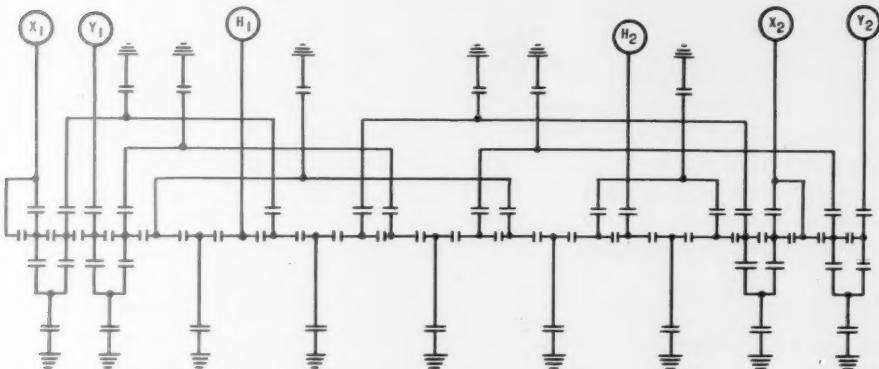


**CALIBRATED POTENTIOMETER** circuit shown measures voltages in terms of percent of the applied voltage. (FIGURE 5)



**METER READINGS** for determining a single T pad equivalent for a more complex circuit are shown in (a). T pad equivalent of

the circuit in (a) is shown in (b). Meter readings are identical with those of the more complex circuit. (FIGURES 6a and 6b)



**EQUIVALENT CIRCUIT** of Figure 3 with five complex meshes replaced by five T pad equivalents is shown above. This simplifi-

cation keeps transformer of Figure 3 within the physical limits of the calculating board shown in Figure 1. (FIGURE 7)

The validity of such an arrangement is illustrated in Figure 4. If a voltage,  $E_0$ , is applied across two capacitors in series (Figure 4a), the voltage across each capacitor is respectively  $E_1$  and  $E_2$ .

$$E_0 = E_1 + E_2 \\ = \frac{1}{\omega} \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

In the above equation it is evident that if resistors are used as in Figure 4b, such that

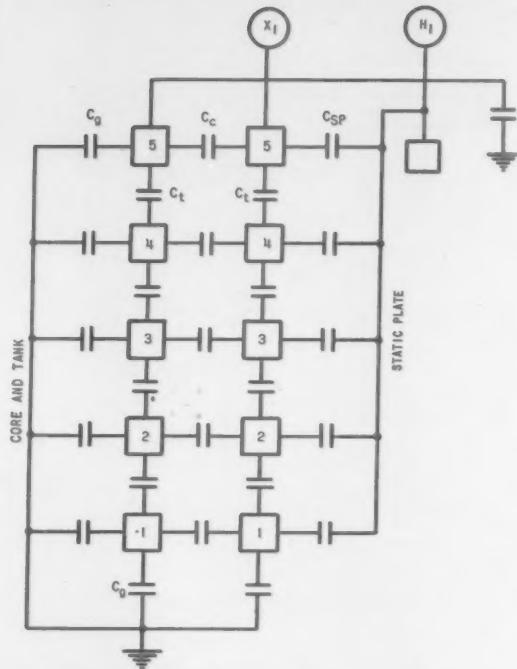
$$R_1 \text{ proportional to } \frac{1}{C_1} \text{ and } R_2 \text{ proportional to } \frac{1}{C_2}$$

the division of voltage will be identical with the capacitance circuit. It is also possible to increase the capacity of any element by simply switching resistors in parallel. There are five ranges of elements used in the calculating board shown in Figure 1. Thus, by arranging and connecting the various ele-

ments into series and parallel circuits and adjusting the direct reading dials, any capacitance network can be simulated for measurement. The physical arrangement of the simulated capacitor boxes is facilitated by using aircraft type slide-snap fasteners on each of the 200 units.

The direct-current voltage source is a metallic rectifier. Voltage is indicated by a calibrated potentiometer, as shown in Figure 5, wherever the probe is placed. The potentiometer is a precision unit and the dial is accurately calibrated in terms of percent of the applied voltage. Balance is indicated by a galvanometer connected between the slider of the potentiometer and the probe. With this instrument the reading will always be accurate in terms of percent of applied voltage regardless of fluctuations in the supply voltage.

When this calculating board was designed, it was made of a size to handle a shell type transformer consisting of either



**DETAIL EQUIVALENT CIRCUIT** of the line end portion of a winding is used to evaluate turn-to-turn voltage stresses. It is also a factor in determining the need for and proper arrangement of the static plate often used to control distribution of voltage stresses. (FIGURE 8)

16 coils in a single group or two groups of eight coils each. Experience indicated that a more intricate circuit than had been previously assumed adequate would be of greater use. This, together with the complexity of modern transformers, has overtaxed the physical limits of the present board.

This has been overcome with a technique of establishing equivalent circuits between designated points in the capacitance circuit to reduce the number of elements necessary. The capacitance network of Figure 3 was handled in this manner by substituting an equivalent network for each of the networks indicated as a, b, c, d, and e. For example, the equivalent for circuit a of Figure 3 is arrived at by setting up the components making up circuit a on the board and applying voltage from the d-c source as indicated in Figure 6a. With the addition of a microammeter and a load, two current measurements and two voltage measurements are taken at points indicated. From these measurements  $C_1$  in the equivalent circuit, Figure 6b, is evaluated as:

$$C_1 = \frac{10^4 (I + I')}{V - V'} \text{ micromicrofarads}$$

In a similar manner as:

$$C_2 = \frac{10^4 (I - I')}{VC_1 - 10^4 I} C_1 \text{ micromicrofarads}$$

Upon evaluating, the five networks of Figure 2 are replaced by their equivalent and the circuit reduces to that shown in Figure 7. This circuit can be readily placed on the calculating board and detail measurements made around the network. If measurements are needed at points within one of the networks replaced by an equivalent circuit, these points can be obtained by using the exact circuits rather than the equivalent circuits for the one network only. If this information is necessary from each of the five regions, it would be necessary to insert the exact circuits, one at a time, and make the necessary readings.

### Board aids designer, protects user

In many transformer designs, readings from the calculating board are used to determine the amount of insulation required in certain areas. This can be done for those parts of a transformer where the maximum stresses are determined by the initial voltage distribution. In other regions where the amplitude of subsequent oscillations determine the insulation requirements, the basic information which permits their calculation is arrived at from the initial distribution obtained on the calculating board.

The circuits considered in Figures 2 and 3 do not give consideration to the turn-to-turn capacity which is important in the design of turn-to-turn insulation and in determining the need for a static plate. Such an analysis is undertaken with a circuit similar to Figure 8 in which each square represents the conductor with capacitors indicated between turns both radially and laterally. In addition, there is capacitance from each coil-to-ground or to other windings. Figure 8 illustrates a detail circuit consisting of the two low voltage coils to the left of the high voltage static plate shown in Figure 2b. Thus, by localizing the circuit, determinations can be made of the initial voltage distribution as fine a detail as is warranted by the problem at hand.

Such a calculating board gives the transformer user additional assurance that he is getting a properly designed transformer for it permits a detailed analysis of many parts under transient conditions, which heretofore could not be done in the time available. Usually it takes about eight hours to arrive at the elemental data necessary before any design can be placed on the calculating board. More complicated units may require little longer calculating. This same elemental data would be used of necessity in any calculation procedure even if the board was not available.

The calculating board is a remarkable time saver in that these elemental units can be connected into a circuit and all essential measurements taken within a day or two compared with months for the calculation of an equal number of points in a circuit similar to that of Figure 3. Mathematical procedures require the reduction of circuits such as Figure 1 or Figure 2 by means of the well-known pi or T equivalents, combining capacitors as substitutions are made, keeping in mind the point of voltage determination. Once the voltage is arrived at for this particular point, the circuit must be reconstructed in order to arrive at the voltages occurring at other points under simultaneous application. This is a long, laborious process.

There are a number of circuit connections used during a complete transformer impulse test. This calculation procedure would of necessity have to be duplicated for each of the critical test conditions used. On the calculating board these various test connections can be made by grounding the appropriate terminals either solidly or through linear resistance and applying voltage to the terminal under test. In this manner all of the test connections can be evaluated as rapidly as the probe can be moved and the potentiometer rebalanced. This simplicity encourages exploration for conditions on other than specified test connections.

The board permits a much more detailed analysis of the phenomena occurring within a transformer during transient conditions than has been practical heretofore. As new problems arise, methods of analysis will undoubtedly be developed that will improve the evaluation of these phenomena still further.

# the BETATRON today

Part One of Two Parts



by R. C. O'DELL  
Betatron Group  
Transformer Section  
Allis-Chalmers Mfg. Co.

## A roundup of theory and application of the 24,000,000-volt betatron from the engineer's viewpoint

In the last six years the betatron has emerged from the laboratory world of cyclotrons, synchro-cyclotrons, betatrons, synchrotrons, bevatrons, and linear accelerators as a practical tool that will have great significance in industry and medicine. Although a number of articles have appeared on various phases of betatron theory and application, much of it has been either oversimplified or else written for those having a post-graduate physics background. The development of the betatron for commercial use in radiography and cancer therapy has been an engineering job. When viewed from this standpoint, the theory, design, construction, and application are of particular interest to the engineer.

Betatrons in use today range in size from 2 to 300 million volts. The higher voltage units have all been built for laboratory research, as was the original 2 million volt machine. The 22-24 million volt betatron has been developed to a high degree of dependability and it is this unit which will be discussed in this article.

### General theory linked to transformer

To produce high speed electrons the betatron employs the magnetic induction effect used in a transformer. In a transformer the primary winding connected to an a-c voltage source establishes a varying flux in an iron core. The secondary winding wound on this core has induced in it a voltage equal to the number of turns in the secondary winding times the rate of change of flux.

In the transformer (as in any piece of electrical apparatus) the electric current which flows is made up of the drift of free electrons present in the wire. Figure 1 shows a transformer in which the secondary is a vacuum tube and which is placed within part of the magnetic field. Note that an air gap exists in the center leg at the tube and that the tube is recessed within the core. This arrangement is necessary to obtain the proper flux conditions inside of and at the location of the electron path. Now, if electrons are injected into this tube as the magnetic field increases, a force will be exerted on the particles that will accelerate them along a circular path.

ELEMENTARY DIAGRAM of betatron shows how electron stream in tube is comparable to secondary winding of shell-type transformer. (FIGURE 1)



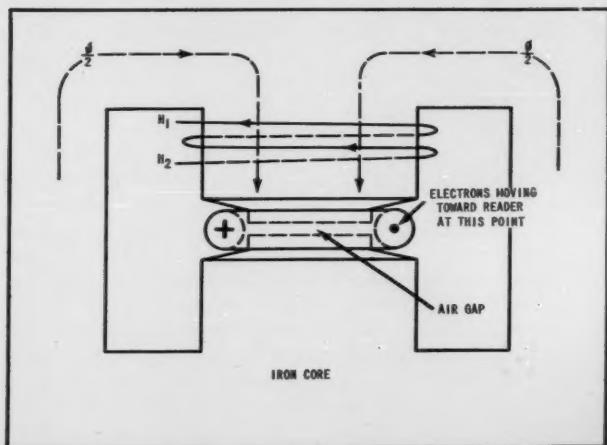
The force acting on the particles is proportional to the rate of change of flux linking the electron path and the magnitude of the field within and at the orbit position.

The energy gain per revolution is equal to the time rate of change of flux,  $\frac{d\phi}{dt}$ , linking the orbit. Since the electrons circle the orbit a great many times before coming to rest, the energy gain is tremendous.

For example, in the 24,000,000 volt betatron, the electrons circle the orbit about 350,000 times and travel a total distance of 260 miles. All of this happens in one-fourth of a cycle, or less than 1.4 milliseconds, since the excitation frequency of the magnet core is 180 cycles per second. The average voltage gain per turn at the orbit is about 70 volts and multiplying this figure by 350,000 gives approximately 24,000,000 volts. (First revolution attains approximately 144 volts and the last zero volts.)

Thus a betatron is similar to a transformer but has an advantage in that it is unnecessary to produce the millions of volts on a secondary coil (with its complexity of insulation problems) and then apply that voltage to a high vacuum X-ray tube in order to accelerate electrons to high energy. The electromotive force is instead continually applied directly to the electron stream. The 24,000,000 volt potential does not exist in any way to cause insulation problems.

The analogy of a simple transformer is an excellent way to show how the betatron works in principle only. However, the parallel cannot be drawn too closely because there must exist a very definite relation between the flux enclosed by the



orbit of the circling electrons in the betatron magnet and the flux density at the orbit. In a transformer it is only necessary that the same flux link both primary and secondary windings. However, the betatron requires critical flux shaping and flux distribution. In the transformer there are wires to contain and guide the electrons, while in the betatron the electrons must be guided by the shaping characteristics of the magnetic field. This is accomplished by placing an evacuated tube in an air gap of certain definite geometry in the betatron magnet core.

### Fundamental equations simple

The complete mathematical theory of the betatron is quite complex; however, the fundamental equations can be derived rather simply. First consider the path of an electron in an evacuated space, passing thru a uniform constant magnetic field. The electron path will be curved in an arc of constant radius by this field. The force that acts on the electron is equal to:

$$(1) \quad F_m = \frac{HQv}{c}$$

where:

$v$  = Speed of electron in cm/sec.

$F_m$  = Force in dynes

$H$  = Field density in gauss

$Q$  = Electron charge in esu

$c = 3 \times 10^{10}$  (a constant of proportionality)

This force is at all times perpendicular to the instantaneous direction of motion. Thus, the path must be the arc of a circle. The force equation just given is also the required centripetal force on the electron which is more familiarly written in terms of mass, velocity, and radius:

$$(2) \quad F_c = \frac{mv^2}{R}$$

$$(3) \quad \text{Then} \quad F_m = F_c = \frac{HQv}{c} = \frac{mv^2}{R}$$

### the BETATRON in INDUSTRY

Standard X-ray equipment (high voltage transformer and X-ray tube) is quite limited in the thickness of steel that can be successfully radiographed. A 400 kv unit is effective up to about three inches, a two million volt unit up to about six to eight inches. For thickness above these limits, the conventional X-ray equipment is not penetrating enough.

In radiography a flaw is seen as the difference in the density of the shadows on a film. The radiograph is a shadow graph rather than a picture produced by reflected light. The "lights and darks" on the film depend upon the difference in density of the material in the path of the X-rays. The denser or thicker the object, the more radiation will be absorbed by it, and less will reach the film.

The high speed of radiography with the 24 million volt betatron compared to other X-ray generators is shown in Figure A. This speed is possible because of the superior transmission characteristics of X-rays through steel at this higher energy. The high point of the transmission curves (Fig. B) occurs at a peak energy of about 22 to 24 million volts. This peak in the transmission curve is the result of the unusual behavior of the absorption coefficient of X-rays for steel as a function of energy. The absorption coefficient has a minimum value between 6 to 8 million volts and is greater

(4) and

$$R = \frac{mv}{QH}$$

(4a) or

$$mv = \frac{HQR}{c}$$

where:

$m$  = Mass of electron in grams

$R$  = Radius of curved path in cms.

A study of this equation will reveal that  $Q$  and  $c$  are the only physical constants and that, if  $R$  is to be constant,  $mv$ , the momentum, and  $H$  must vary in the same ratio. Electrons traveling at a high velocity are necessary to produce high energy X-rays. Therefore, it is necessary to find some means of increasing the field density and electron momentum the same percentage, otherwise an X-ray tube with a fixed orbit radius is not possible.

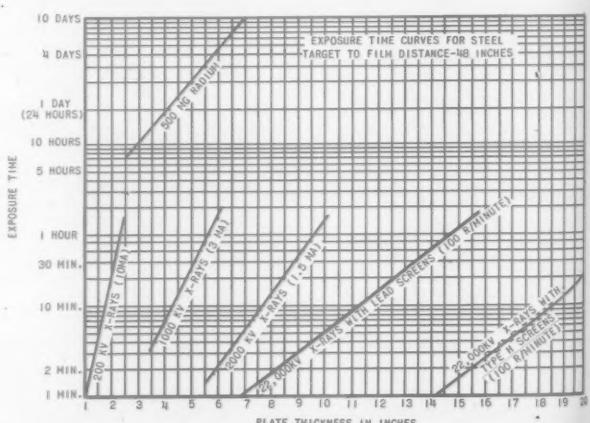
The only effect of a constant magnetic field is to deflect the electron since the force is always perpendicular to the direction of motion. The electron speed is unchanged. Now, the action of a time-varying magnetic field on the electron will be evaluated.

Suppose we were to consider a wire placed in a position coincident to the electron orbit. There would be an emf induced in the wire due to the time rate of change of flux. Since the wire is simply a carrier of electrons, we could take it away if we could provide forces to maintain the electrons in the same orbit. Assume for the moment that this has been done. Now the emf still existing in space will act directly on free electrons. This is in such a direction as to accelerate the electrons opposing the magnetic field which sets up the flux condition. This is the familiar Lenz's Law.

Faraday's law states that the electromotive force induced by a changing magnetic field through a fixed circuit is equal to the time rate of change of flux.

$$(5) \quad \text{Emf per revolution} = -\frac{d\phi}{dt} \cdot \frac{1}{c} \quad (\text{in esu})$$

above and below this range of energy. The spectrum of the X-ray energies from a 24 million volt betatron is such that the average energy is in this minimum absorption range. Thus optimum penetration is achieved when operating the betatron



TIME REQUIRED for X-raying steel of various thicknesses shows the advantage of the betatron in thick section radiography. (FIGURE A)

This law also states that the line integral of the electric intensity around any fixed closed curve is equal to the time rate of change of flux through the curve.

Therefore

$$(6) \quad \int E \cdot ds = -\frac{d\phi}{dt} \cdot \frac{1}{c}$$

Assuming  $E$  to be constant for the duration of one revolution

$$\text{Emf per revolution} = \int_0^{2\pi R} E \cdot ds = E \cdot 2\pi R = \frac{1}{c} \cdot \frac{d\phi}{dt}$$

$$(7) \quad \text{then } E = \frac{1}{2\pi R c} \cdot \frac{d\phi}{dt}$$

Where:

$E$  = Electric field intensity or force per unit charge in esu.

The force acting on the electrons can be expressed by the following equations:

By definition:

$$(8) \quad F_T = EQ$$

$$(9) \quad F_T = \frac{Q}{2\pi R c} \cdot \frac{d\phi}{dt}$$

Where:

$F_T$  = Tangential force, in dynes, acting on the electrons.

Two conditions have now been established. The first is that a uniform magnetic field will force a moving electron to travel in a circular path. The second is that a time-varying magnetic field will create a force which will act directly on the electron to accelerate it.

Up to this point we have assumed that the electrons moved in an orbit of constant radius,  $R$ . Now with the two foregoing conditions established, a flux arrangement may be set up to insure this.

According to Newton's second law of motion, force is equal to the time rate of change of momentum.

$$(10) \quad \text{Thus } F_T = \frac{d(mv)}{dt}$$

$$(11) \quad \text{and from above } F_T = \frac{Q}{2\pi R c} \cdot \frac{d\phi}{dt}$$

$$\frac{d(mv)}{dt} = \frac{Q}{2\pi R c} \cdot \frac{d\phi}{dt}$$

Then integrating the above

$$(12) \quad mv = \frac{Q}{2\pi R c} (\phi - \phi_0)$$

(13) If the electrons are accelerated from time  $\phi_0 = 0$  then,

$$(14) \quad mv = \frac{Q}{2\pi R c} \phi$$

and since we have already shown

$$mv = \frac{HQR}{c} \quad (\text{see equation 4a})$$

then

$$(15) \quad \phi = 2\pi R^2 H$$

where  $\phi$  = The number of lines of magnetic flux enclosed by the orbit during the accelerating period.

$H$  = Field density in gauss at orbit.

This last equation means that the total flux within the orbit must be twice that which would exist if the magnetic field were constant in space. To satisfy this requirement one must have a strong central field to supply the necessary flux to accelerate the electrons and a weaker field at the orbit to hold them in place. The practical way to obtain this flux relation is to provide an air gap with less reluctance within the orbit than at the orbit.

### Electrons oscillate around orbit

Figure 2a shows the magnetic field-time curve. The electrons are injected into the tube near time for  $\phi = 0$  and are allowed to accelerate until  $\phi = \phi_{\max}$ . At this point the delicate flux

at a peak energy of 24 million volts.

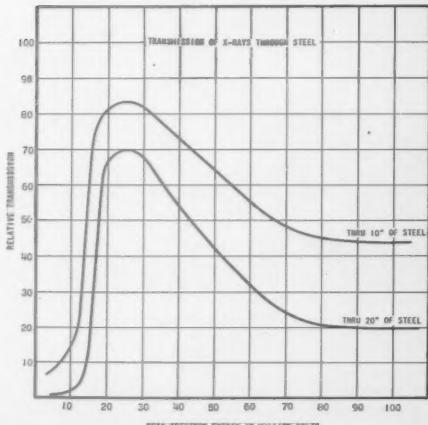
A further advantage of this energy range is the characteristic of extremely wide latitude. This reduces the total number of radiographs required for the inspection of complicated

machines such as electric motors, gasoline engines, valve bodies, etc. It is possible to see a flaw in a steel forging or casting throughout a section varying from 12 to 18 inches with a single-exposure a latitude of 6 inches — on a single film.

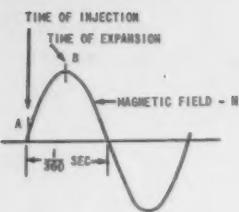
Flaws  $\frac{1}{32}$ -inch deep can be seen in steel sections from 3 to 12 inches thick and  $\frac{1}{16}$ -inch deep in pieces up to 20 inches thick. These flaws may be as narrow as .005 inch.

Where minute flaws or extremely close assembly tolerances may exist, the small focal spot (.005-inch  $\times$  .010-inch) in the target of the betatron X-ray tube makes possible the enlargement of a minute flaw by simply increasing the specimen to film distance. The small focal spot, inherent in the betatron, will yield a sharply defined image. A gap as narrow as .001-inch parallel to the plane of the X-ray beam is detectable in this manner.

Application of the betatron in industry is not limited to inspection of castings and forgings. In one installation, used transmissions and motors are being checked for internal condition and alignment of parts to determine the extent of reconditioning required. In others, internal mechanisms of military equipment are checked before shipment, and consideration has been given to the possibility of high speed X-raying of hot billets before cropping, saving steel mills needless reworking of thousands of tons of steel yearly.

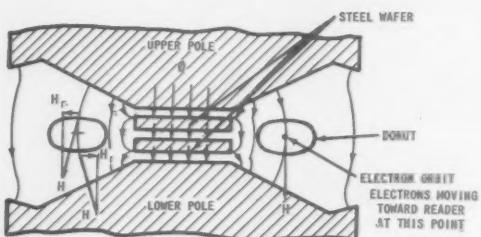
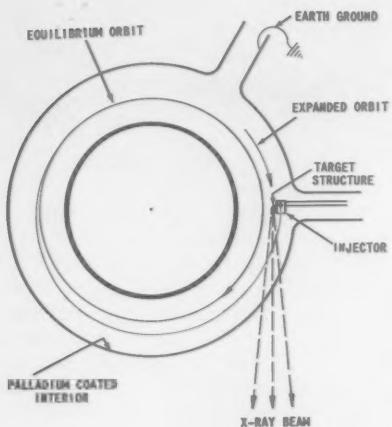


RELATIVE TRANSMISSION as a function of peak betatron energy shows best penetration in the 22- to 24-million-volt range. (FIGURE B)

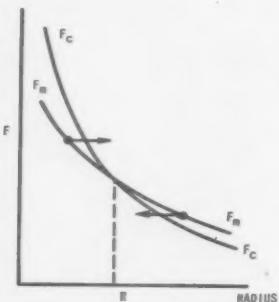


MAGNETIC FLUX as a function of time is plotted at left. (FIGURE 2a)

TOP VIEW of X-ray tube shows injector, target, and electron path. (FIGURE 2b)



LINES OF FORCE between betatron poles follow pattern shown. Radial component  $H_r$  forces axially oscillating electrons back toward median plane, and away from tube walls. (FIGURE 3)



CENTRIPETAL FORCE  $F_c$  is required to hold an electron at radius  $R$ .  $F_m$  is the magnetic force supplied to the electron. Radial oscillation occurs about point  $R$ . (FIGURE 4)

conditions are upset (by a method to be described in Part II) and the electrons spiral outward to the target. In Figure 2b is shown a top view of the X-ray tube or donut depicting the electron path and resultant X-ray beam.

Now if one could be sure that the electrons would stay in the equilibrium orbit until they are forced to spiral outward to strike the target, the problem would end there. But the effects of scattering by the residual gas in the tube and of the space charge in the electron stream mean that strong focusing forces must be used to hold the electrons in the orbit.

Any axial displacement can be reduced by providing a bulging magnetic field. Figure 3 shows the curvature of the lines of force between the betatron poles. If an electron leaves the plane of the equilibrium orbit, it finds itself in a magnetic field with a slight radial component. This radial component on the top side is oppositely directed from that on the bottom side of the orbit plane. This always forces the electron back toward the median plane. The pole face is tapered away from this plane to produce the bulging field.

The radial displacements that occur are reduced by radial focusing forces supplied by the radial distribution of the field. An electron traveling in a circle requires a centripetal force  $F_c = \frac{mv^2}{R}$ , the force varying inversely with the radius. The equation for the force exerted by the magnetic field on the electron is  $F_m = \frac{QHv}{c}$ . If  $H$  can be made to vary less rapidly, than inversely with  $R$ , then for radii greater than the desired radius for the orbit, the field will supply a force greater than the  $F_c$  required to hold the electron in a circular path, and the electron will spiral back toward the equilibrium radius. At a radial position less than the desired radius,  $H$  will supply less than the desired magnetic force to the electron and the electron will spiral outward to an equilibrium radius. This may be illustrated by Figure 4.

Accordingly, there are forces established suitable for the radial oscillation of an electron across the equilibrium radius. Thus, the magnetic field should decrease less rapidly than  $\frac{1}{r^n}$  and it should also decrease with increasing radius so that the bulging lines of force will provide axial focusing. Radial damping will occur provided that the magnetic field decreases with radius as  $\frac{1}{r^n}$  where  $n$  lies between  $\frac{1}{2}$  and 1. Axial damping, as well as further radial damping occurs with increasing field strength,  $H$ , as a function of time. Without damping relatively few of the oscillating electrons could be made to strike the target at the end of the accelerating period. This would result in poor X-ray yield.

Since the damping force is relatively large when  $H$  is small, the electrons can be injected from a point at a radius slightly greater than the equilibrium orbit and not strike the injector again on one of the first few revolutions. This makes possible the process which is used to get the electrons started.

### Electron velocity nearly speed of light

During the development of these equations, a constant electron mass was assumed. This is approximately true when velocities much smaller than light are involved. However, in accordance with relativistic theory, as an electron's velocity increases, its mass also increases. Thus the mass must be defined in terms of its velocity. The derivations made are



**FIVE MINUTES** is the time required to provide a radiograph of the induction motor. Clearances and dimensions in the air gap, bearings,

core steel, and windings will all be clearly indicated. The 24-million-volt betatron lends itself to periodic inspection of heavy equipment.

correct and do not conflict with relativistic theory because the momentum of the particle (mass  $\times$  velocity) and its time rate of change were considered rather than mass alone.

It may be shown that the kinetic energy of an electron at relativistic speeds is:

$$(16) \quad KE = m_e c^2 \left[ \frac{1}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} - 1 \right]$$

and

$$(17) \quad m = \frac{m_e}{\sqrt{1 - \left( \frac{v}{c} \right)^2}}$$

where

$m_e$  = Rest mass of the electron in grams

$m$  = Relativistic mass at speed  $v$  in cm/sec.

$$\therefore mv = \frac{m_e v}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} = \frac{HQR}{c} \quad (\text{from 4a})$$

$$(18) \quad \text{Thus } KE = \frac{c}{v} HQR - m_e c^2 \quad \text{ergs}$$

Since at the energies we are concerned with (24 million volts) the electron velocity is greater than 99% of the velocity of light, this equation may be reduced to:

$$(19) \quad KE \text{ in million electron volts (MEV)} * \cong 3 \times 10^{-4} HR - .5$$

\* Common usage has shortened the expression of million electron volts to million volts for the available kinetic energy of a particle accelerator since the two are numerically equal.

### Energy limited by core saturation

If the two to one flux relation given in Equation 15 is evaluated in the light of the saturation point of core steel, a practical limit to the maximum energy of electrons obtainable with betatrons can be determined.

The wafer (see Figure 3) will start to saturate at about 12,000 gauss. However, since it is not desirable (in fact, impossible) to completely fill the area enclosed by the equilibrium orbit with steel, the average density within the orbit is about 10,000 gauss. Then, with the field at the orbit position being 5,000 gauss, equation (19) reduces to:

$$(20) \quad MV \cong 1.5R - .5$$

With the orbit radius of the 24 million volt betatron about 19 cm the theoretical maximum energy obtainable with the machine is thus approximately 28 million volts. If attempts were made to operate in the saturation region the two to one ratio would be destroyed, since the flux density at the orbit would increase faster than that within the orbit, and the electrons would spiral inward to be lost on the tube walls.

The limitation in obtaining very high energies with a straight betatron (for which the foregoing equations apply) is physical size since the energy output is almost directly proportional to the orbit radius. Part of this same limitation is in the power requirements for such a machine. With units above 70 million volts using betatron principles, additional techniques are required. Such machines are called flux biased betatrons or synchrotrons.

\* Design, construction, and electronic circuits of the 24-million-volt betatron will be discussed in the Fourth Quarter, 1950, issue of the ELECTRICAL REVIEW. Also included will be up-to-date reviews of the betatron's contributions in cancer therapy and in research.

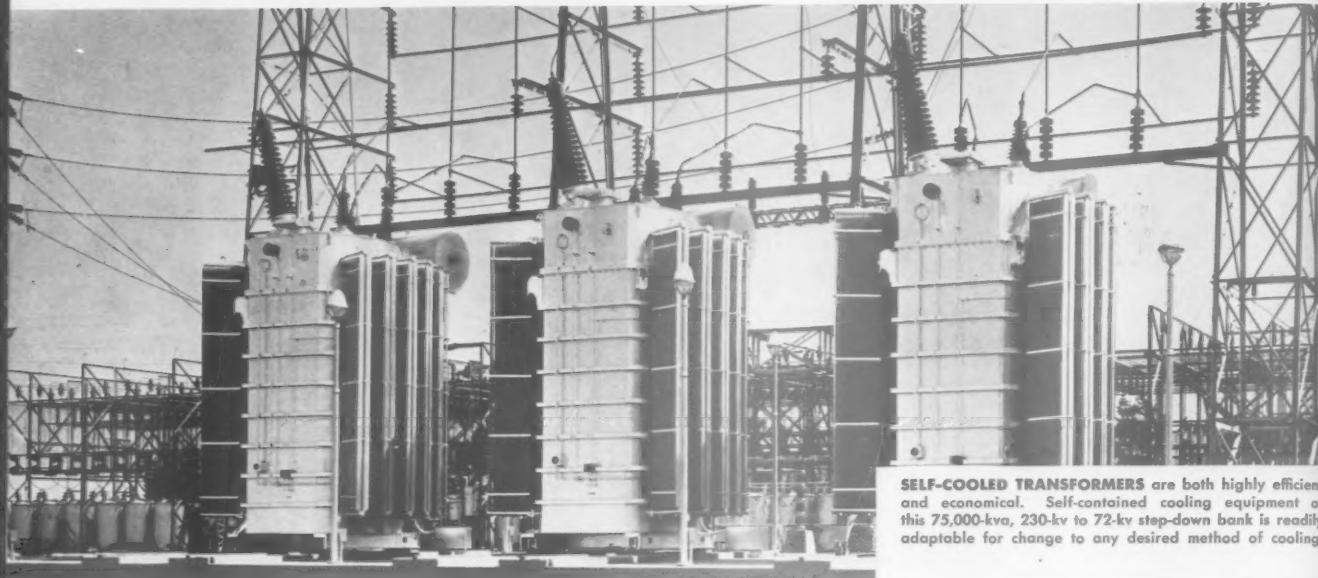
**SPIRAL CASING SECTIONS** for the first of four 105,000-hp, 400-ft head, 180-rpm single vertical welded plate of steel spiral casing hydraulic turbines for the Bureau of Reclamation's Hungry Horse power plant in Montana are shown being prepared for machining. To aid shipment, casings are built in four such sections of approximately 60,000 pounds each.

Allis-Chalmers Staff Photo





# Trends in Power Transformer Application



**SELF-COOLED TRANSFORMERS** are both highly efficient and economical. Self-contained cooling equipment of this 75,000-kva, 230-kv to 72-kv step-down bank is readily adaptable for change to any desired method of cooling.

by

**L. W. SCHOENIG**  
Transformer Section  
Allis-Chalmers Mfg. Co.



*Significant operating characteristics as well as initial cost should be considered when selecting a large power transformer*

**ECONOMICAL APPLICATION** of transformers is a very important part of electrical system engineering.

Considerable research is done by most utilities to determine the proper transformer for each application. Many factors, such as continuity of service, load factor, losses, initial cost, evaluation of losses, etc. are involved, and each must be considered fully before a definite selection is made.

The type of transformer applied to power systems has changed considerably in the last 20 or 30 years. Early transformers were usually single-phase and self-cooled, since alternate methods of cooling were not available and transformer design had not yet reached the point where it was desirable to use three-phase in lieu of single-phase transformers. Failures sometimes occurred because the transformer designer was handicapped by the limited available materials, technical knowledge and working tools.

Modern transformer design and materials have changed the situation entirely. The impulse surge generator and the oscilloscope as working tools, plus a better understanding of insulation fundamentals, have enabled engineers to develop electrically better and stronger insulation assemblies, to check

voltage distribution under impulse conditions, and to check the workmanship of assembled transformers. As a result transformer failures from lightning, switching surges, or other reasons are practically unknown today.

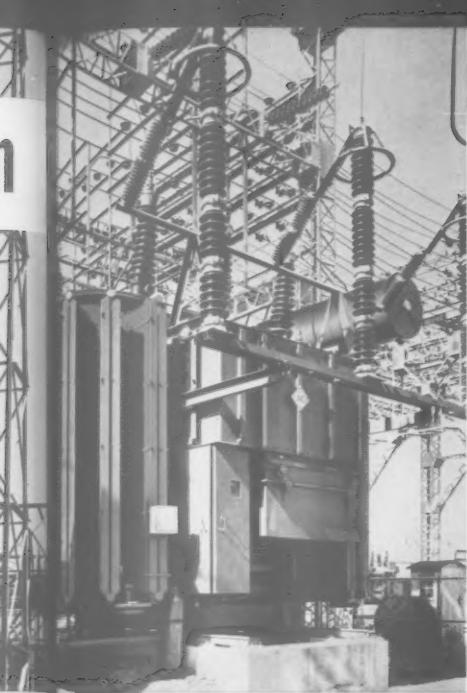
## Cooling practices differ

The heat generated in a transformer can be dissipated by using air or water in a number of different ways.

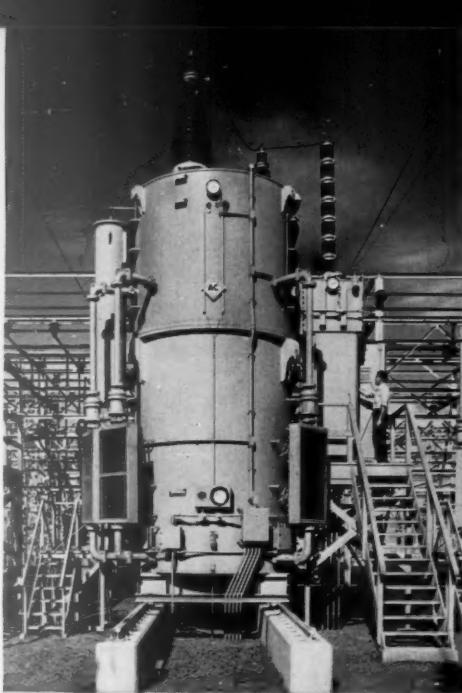
Power engineers are familiar with the so-called self-cooled transformer which has large tube type radiators through which oil circulates by natural convection, and which cool the oil by the natural flow of air over the external surfaces. The amount of heat dissipated can be increased materially by increasing the flow of air over the tube type radiators. This can be conveniently accomplished by use of fans or blowers. Consequently, transformers with this type of cooling equipment are called "forced-air-cooled." The capacity of transformers can be increased about one-third by the use of this equipment. The heat dissipated can be further increased by circulating the oil through radiators with pumps in addition to blowing air over the radiator. To facilitate the transfer of heat, the area of flow for the oil and air is made very concentrated. An automobile type radiator is usually used so that the air flow may be concentrated over a small area thereby obtaining very efficient heat transfer. Such highly efficient cooling equipment increases transformer capacity by about 67 percent.

The self-cooled transformer has the advantage of being completely self-contained and requiring no additional cooling equipment. Forced-air-cooled and forced-oil-cooled units, on the other hand, require a reliable power supply for the fans and pumps.

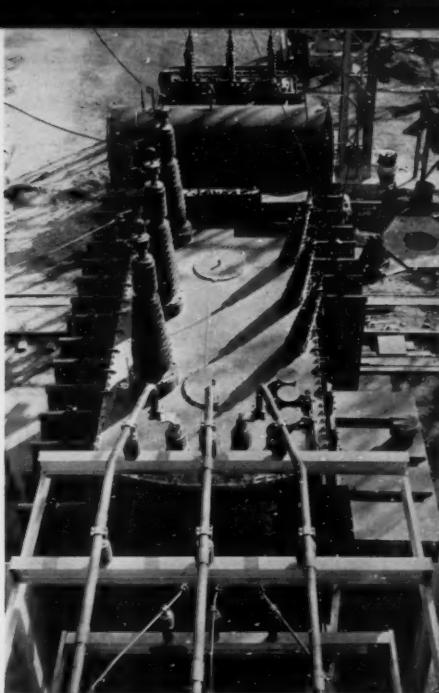
In all probability, future expansion for a power supplier can be provided for best by self-cooled transformers arranged for future addition of forced-air fans or forced-oil cooling equipment. The capacity can be increased from one-third to



**BIGGER LOADS** can be carried by transformers built with forced-air fans. This 24,000/30,000-kva, 3-phase OISC/FA transformer shifts power to three circuits.



**FORCED-OIL-COOLED** transformers provide considerably greater capacity and sizable savings in weight and floor space. This 50,000-kva, 230/115-kv autotransformer is representative of forced-oil-cooled construction.



**THREE POWER SYSTEMS** are tied together efficiently with this 30,000/40,000-kva 161/138-69-13.2-kv, 3-phase transformer recently installed by a midwestern utility.

two-thirds by the addition of such cooling equipment. The cost increase for future forced cooling is nominal and should be considered if future expansion is contemplated.

The most common method of using water as a cooling medium is with cooling coils located inside the transformer tank in the path of the hot oil. The heat generated in the transformer is transferred from the oil to the water which is circulated through the copper cooling coils. The heat transfer from oil to water can be more efficiently accomplished by pumping the transformer oil through external heat exchangers. Transformer capacities can be increased about 67 percent with this type of cooling equipment.

The use of water as a coolant is determined by its availability. Transformers located near dam sites or steam generating stations have the choice of air or water, whereas most step-down transformer stations or substations are limited solely to air for the cooling medium.

### Insulation level varies with application

The subject of basic insulation levels has been discussed quite extensively for some time. Many engineers are of the opinion that the basic insulation levels of higher voltage transformers can be safely reduced one class below standard if proper protection is provided. The contention is substantiated by the practice during the last few years of using reduced insulation (lower basic insulation levels) on transformer windings 115 kv and higher. In other words, some transformers used on 138-kv systems have had high voltage windings insulated for 115 kv; on 230-kv systems, 196-kv insulation, etc.

Reduced insulation, in addition to resulting in lower initial cost, lowers impedance and losses since the transformer can be made with less insulation between groups of windings, from windings to ground, and on individual turns. Impedance is from five to ten percent lower for a reduction of one insulation class. With normal design the losses of a transformer with

reduced insulation of one class are about 2½ to 5 percent lower.

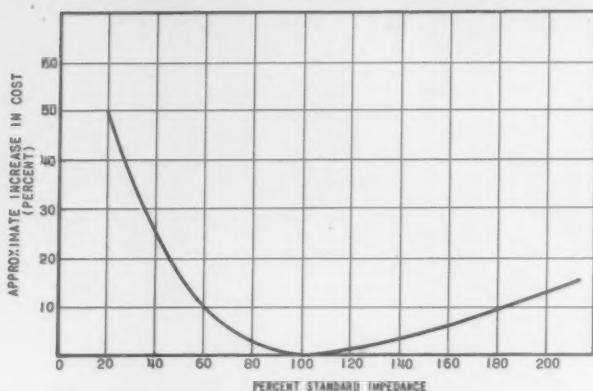
Reduced insulation is usually incorporated in transformer windings which are wye-connected and solidly grounded since voltage surges due to impulse or switching can be limited to materially lower magnitudes on solidly grounded systems than on ungrounded systems. Another factor influencing the use of transformers with reduced insulation on wye-connected solidly grounded systems is the relatively high protection ratio obtained. The protection ratio is the ratio of transformer basic impulse insulation level to lightning arrester protection level. Reduced insulation has been used on delta-connected windings, but the practice is not generally followed because the reduction in protection usually does not conform to good operating practice.

It is desirable, on transformers using reduced high voltage insulation, to mount properly coordinated lightning arresters as near the bushings as possible to limit the voltage increase between the lightning arresters and the transformer, under impulse conditions, to as small a value as possible.

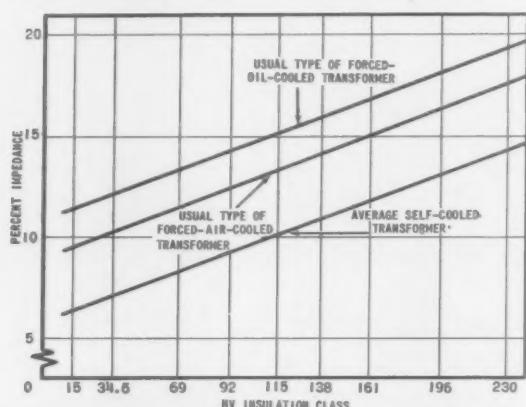
### Planning, equipment improve service

The service reputation which utilities enjoy today is the result of sound utility planning which manufacturers assisted by designing more reliable equipment. Today, transformer failures are the exception rather than the rule. Impulse-proof insulation assemblies, vacuum filling, better understanding of heat transfer in transformers, and many other items have contributed to greater reliability.

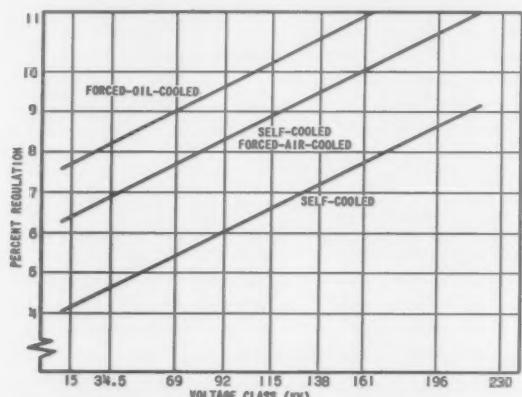
Early ideas of transformer application were that single-phase, self-cooled transformers were the only way that continuity of service could be obtained, since self-cooled transformers required no auxiliary equipment and single-phase units enabled spare capacity to be obtained economically and conveniently. Forced-cooled transformers had no place on early



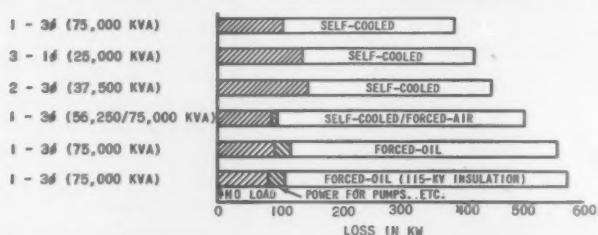
**TRANSFORMER COSTS RISE** when non-standard impedances are specified. Approximate cost curves indicate that subnormal impedances raise transformer costs more sharply than those above normal. (FIGURE 1)



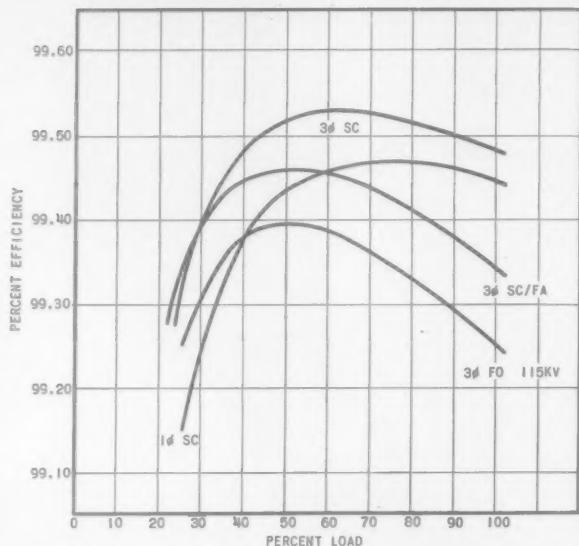
**COOLING AND INSULATION LEVEL** affect transformer impedance. Systems requiring high impedance transformers are ideal for forced-oil cooling. Self-cooled units fit low impedance needs. (FIGURE 2)



**COMPARISON OF AVERAGE REGULATION** at 0.8 power factor for self-cooled, regular forced-air and forced-oil-cooled transformers indicates that forced-oil-cooled units need the most regulation. (FIGURE 3)



**TRANSFORMER LOSSES** for various methods of handling a 75,000-kva, 138-kv, 3-phase load are compared to show the no-load loss, total loss, and power requirements for necessary cooling equipment. (FIGURE 4)



**EFFICIENCY CURVES** for various methods of transforming a 75,000-kva, 138-kv, 3-phase load are shown above for comparison. (FIGURE 5)

power systems. Today, the situation is almost completely changed. The turning point occurred during the war years, 1942 to 1945, when the shortage of materials dictated the use of three-phase forced-cooled transformers on many systems. These transformers worked—and worked well.

The reliability of water-cooled or forced-cooled transformers using oil-to-water heat exchangers depends upon the adequacy of the supply of cooling water, and also the pumping equipment and its power supply. In most cases, water is used for cooling in areas where water is adequate. Obtaining reliable pumping equipment is a minor matter.

On the other hand, the reliability of forced-cooled transformers using air as a cooling medium presents no problem, since it is not too difficult to obtain a reliable power supply for the fan motors and pumps.

Modern transformer core and coil construction, cooling equipment bushings, filling with oil under vacuum, and other advancements are such that the continuity of service obtained with three-phase forced-cooled transformers is equally as good as for single-phase self-cooled transformers.

### Functional needs affect design

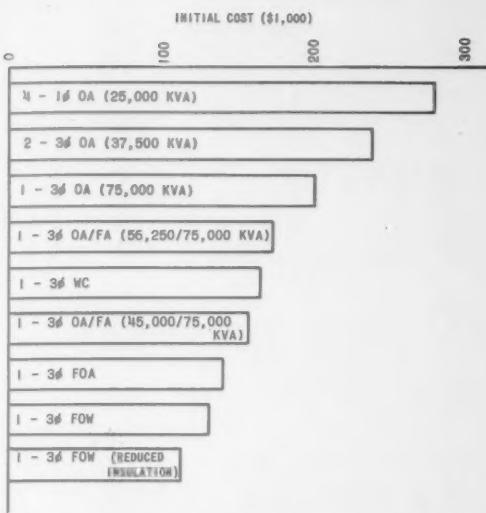
#### Weights and dimensions

The self-cooled transformer, although it has the advantage of being completely self-contained, has the disadvantage of being the largest in weight and dimensions, and the most expensive. The forced-oil-cooled unit, because of the more efficient heat

TABLE 1

	Self-Cooled	Self-Cooled Forced Air	Forced Oil
Total Weight .....	100%	90%	70%
Floor Area .....	100%	90%	70%
Overall Height .....	100%	95%	90%

**COMPARISON OF WEIGHTS** and dimensions for the various ways of handling a 75,000-kva, 138-kv to 13.8-kv, 3-phase load is shown above. Comparison shows that forced-oil cooling has definite advantages over other methods when floor space is a limiting factor.



GRAPHICAL COMPARISON of initial transformer costs for various methods of handling a 75,000-kva, 138-kv load are shown. (FIGURE 6)

transfer has the smallest floor area and weight, and is considerably cheaper. Table 1 compares weights and dimensions for the various methods of handling a 75,000-kva, 138-kv to 13.8-kv load. This comparison indicates that a considerable reduction in dimensions and weights can be obtained by the use of forced cooling.

In larger kva and higher voltage ratings, the shipping limitations may dictate the transformer selection. These restrictions quite frequently require the selection of forced-cooled units.

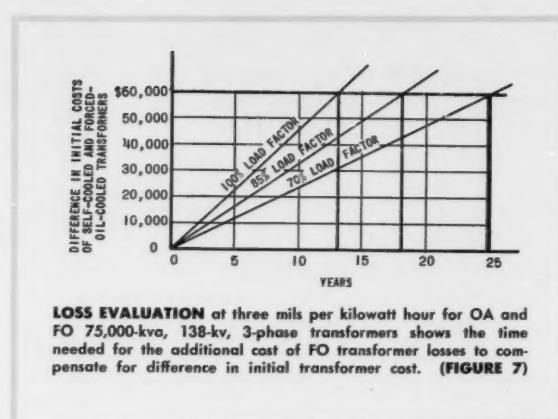
#### Impedance

Interconnected power systems require that considerable thought be given to transformer impedance. New transformers which will operate in parallel with existing units should, of course, have the same percentage impedance, so that all transformers can be used economically at their maximum capacities. However, in many cases, where no parallel operation is required, the desirable impedance of new transformers is determined by calculating board studies.

Unless parallel operation or calculating board studies require special transformer impedances, normal values should be specified, as non-standard impedances affect cost, physical dimensions, and weights. Figure 1 shows the approximate cost increase for non-standard impedances. Note that impedances lower than normal have considerably greater effect on transformer cost than those which are above normal. Non-standard impedances affect weights and dimensions by approximately the same percentages as the cost.

Another significant fact is that a transformer's percent impedance or impedance voltage fluctuates directly with load variations. If the load on a transformer is increased 25 percent, the impedance volts will also be increased 25 percent. It is for this reason that the normal impedance of a forced-cooled transformer at the maximum rating is from 25 to 60 percent greater than the impedance of a self-cooled transformer of comparable rating.

Transformer insulation level and method of cooling have a material effect on transformer impedance. Figure 2 shows the



LOSS EVALUATION at three mils per kilowatt hour for OA and FO 75,000-kva, 138-kv, 3-phase transformers shows the time needed for the additional cost of FO transformer losses to compensate for difference in initial transformer cost. (FIGURE 7)

effect of high voltage insulation level and cooling method on transformer impedances. As shown by the curve, the normal impedance of a transformer increases with the insulation level and with the forced methods of cooling.

#### Regulation

The difference between the actual full-load and no-load secondary voltage of a transformer with the impressed voltage held constant is what determines transformer regulation. The voltage difference increases as the load is increased from zero at no load to an appreciable value at full load. Transformer resistance, reactance and load power factor each contribute to the internal voltage drop. Transformer regulation increases with reactance and resistance, and generally increases materially as the power factor of the load decreases. Figure 3 compares transformer regulation at 0.8 power factor for self-cooled, self-cooled/forced-air-cooled, and forced-oil-cooled transformers.

#### Losses

A system load can be handled among other methods by either three single-phase or one three-phase self-cooled transformer. The combined copper losses of the single-phase units will be the same as for the three-phase transformer. The core loss, however, will be about 20 percent higher for the single-phase units because of the less efficient use of core steel.

The three-phase self-cooled and the water-cooled transformers have the lowest total loss and highest core loss of any cooling method. The forced-oil-cooled unit, on the other hand, has the highest total loss and lowest core loss.

High total loss and low core loss of the forced-cooled transformers result from the increase in capacity with the cooling equipment. The core and coils are actually designed for a load of about 60 percent of rated load. Therefore, the copper loss at rated load is  $(1.67)^2$  times that at the 60 percent rating. The core loss is low as a result of the small core and coils.

Figure 4 compares the no load and total losses of the various methods of transforming a 75,000-kva load from 13.8 kv to 138 kv. Power requirements for fans and pumps have been included. The efficiency curves in Figure 5 were calculated from the losses in Figure 4 by using the following equation

$$\% \text{ efficiency} = \frac{\text{kW load}}{\text{kW load} + \text{losses}}$$

#### Initial cost can be reduced

As an example of transformer initial cost and its relation to the various types of cooling, let us examine the first cost of the different methods by which a given load can be handled.

As a basis of comparison, let us assume that 75,000 kva will be generated at 13.8 kv and transmitted at 138 kv. The low voltage will be delta-connected and the high voltage wye-connected arranged for solidly grounded operation. Figure 6 compares the first cost of some of the various methods of transforming 75,000 kva. Figure 6 shows that the initial cost of a transformer bank can be reduced materially by utilizing three-phase instead of single-phase transformer construction. The initial cost can be reduced further by taking advantage of the various types of cooling equipment and also by utilizing the saving afforded by reducing the insulation level of the high voltage winding.

In the selection of transformers, it may be desirable to temper initial cost by loss evaluation which places a monetary value on each kilowatt of loss. This can be done in many different ways depending upon such items as type of generation—steam, diesel, or hydro, generator and transmission line capacity, type of transformer—step-up or step-down, and load factor.

Probably the simplest method of loss evaluation is to use commonly accepted values such as \$200 per kilowatt of no-load loss and \$100 per kilowatt of load loss. Another simple method is to evaluate the losses at from three to seven mils per kilowatt hour for the life of the transformer.

Figure 7 evaluates losses at three mils per kilowatt hour and compares self-cooled and forced-oil-cooled transformers at various load factors. It is evident that the initial cost advantage of the forced-oil-cooled transformer is offset by the three mil loss evaluation in 13 years for the 100 percent load factor.

#### **Consider "hidden" costs, too**

The type and efficiency of the generating station influences evaluation. A generating cost of from three to five mils per kilowatt hour is usually used for steam stations and seven mils per kilowatt hour for diesel stations. On most hydro systems, however, the cost of generation is usually neglected. Both methods of evaluating consider only the cost of generating the losses and do not take into consideration the loss of revenue energy. This loss of revenue energy affects the evaluation of base load station transformers, other transformers at peak load, and step-down transformers where the transmission line is loaded to capacity. To get a more complete picture, the loss of revenue energy should be evaluated at about 1.8 cents per kilowatt hour and the cost of installing additional generating capacity from \$150 to \$400 per kilowatt.

Evaluation of losses is complex because so many variables affect the final result. Loss evaluation will differ for each system and, for that matter, for each part of a system. As a result, each power supplier must determine the capitalization values which will apply to the part of his system in which the transformer will be used.

Proper application of large power transformers requires that, in addition to first cost, consideration be given to methods of cooling, insulation level, impedance, losses, loss evaluation, and weights and dimensions. The system engineer must determine the amount of emphasis each item is to be given since a transformer which is the proper selection for one application may not be the same for another. Every transformer application must be carefully studied to insure satisfactory and economical operation.

# SOUND ABSORPTION OF PANELS

by

**B. D. NAWN and D. F. THOMPSON\***

**Motor-Generator Section  
Allis-Chalmers Mfg. Co.**

*Test set-up reveals discrepancies between actual and theoretical sound attenuation of various materials considered*

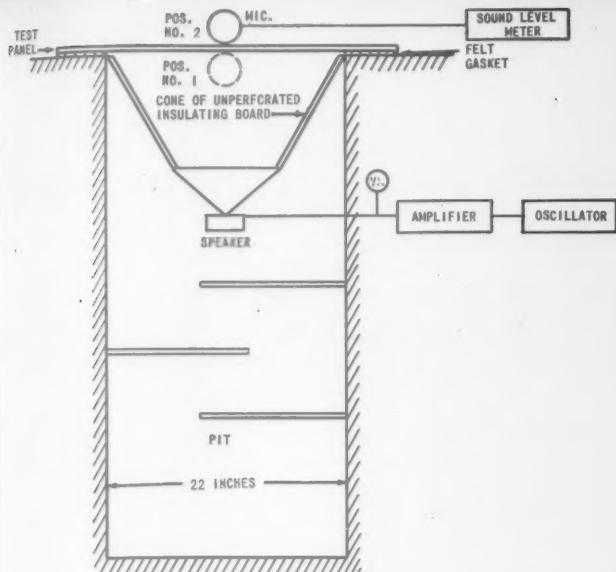
ERY OFTEN in acoustical studies the engineer is forced to rely solely on derived formulas in predicting performance. Actual results, unfortunately, are not always as good as these expressions lead him to anticipate. An almost everyday problem, for example, is the question: "How much can the noise level be reduced if the apparatus is totally enclosed in a mechanically separate housing?" The tests discussed here, which are part of a long range study in noise reduction, were conducted to obtain definite information on quantitative values for the attenuation of sound through panels of various materials as well as through several combinations of these panels. Results show that sound absorption depends almost entirely on (1) the wall weight per sq. ft., (2) the number of completely isolated layers through which the sound must pass, and (3) the absence of all small cracks or openings.

#### **Test methods simple but effective**

The test setup is shown in Figure 1. A seven-foot deep pit beneath a concrete floor served as a relatively soundproof chamber. Access to the pit was through a 22-inch circular opening in the floor. The sound source was a speaker fed by an oscillator through an amplifier. A voltmeter was placed across the speaker voice coil in order to be able to repeat a sound level at a particular frequency. A speaker was suspended in the pit with the front surface of the speaker enclosure 17½ inches below the surface of the floor. The sound seal was accomplished with a ½-inch felt gasket. The felt gasket had an inside diameter of 22 inches and an outside diameter of 34 inches.

The same sound level meter and microphone were used for readings below and above the test panel. With the microphone located beneath the test panel, the speaker volume was set to

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**IN TEST SET-UP** the brick-lined pit in the concrete floor served as a relatively soundproof chamber. Baffles reduced the effects of standing waves. A slight clearance was maintained between the microphone and the panel to prevent mechanical vibration of the microphone. (FIGURE 1)

give approximately 115 decibels. For each 100 cycles from 400 cycles to 1,500 cycles, the sound level meter reading, the frequency, and the speaker voice coil voltage were recorded. Next, the microphone was placed above the test panel. Using the previously recorded voltage for each frequency, the procedure was repeated in order to obtain the corresponding sound level meter readings above the test panel. The difference between corresponding sound level meter readings gave the attenuation across the test panel.

The distance from the center of the microphone to the surface of the panel was slightly over an inch, corresponding to about three percent of a wave length at the lowest test frequency and about 11 percent of a wave length at the highest test frequency.

In practically all cases the sound levels were well above the ambient level of the room. In a few instances where this was not so, the readings were corrected by means of standard curves for this purpose.

The results of all of the tests are shown in Figure 2 plotted as db drop through the panel vs. frequency in cycles per second. For each material, the theoretical attenuation is also indicated by a dashed curve. This theoretical attenuation in sound was obtained from the formula by Rayleigh:

$$\text{db drop} = 10 \log_{10} \frac{1}{1 - \frac{\pi \times D_1 \times h/D \times L}{\sqrt{1 + (\pi D_1 \times h/D \times L)^2}}}$$

Where:  $D$  = density of air

$D_1$  = density of panel material

$h$  = thickness of panel

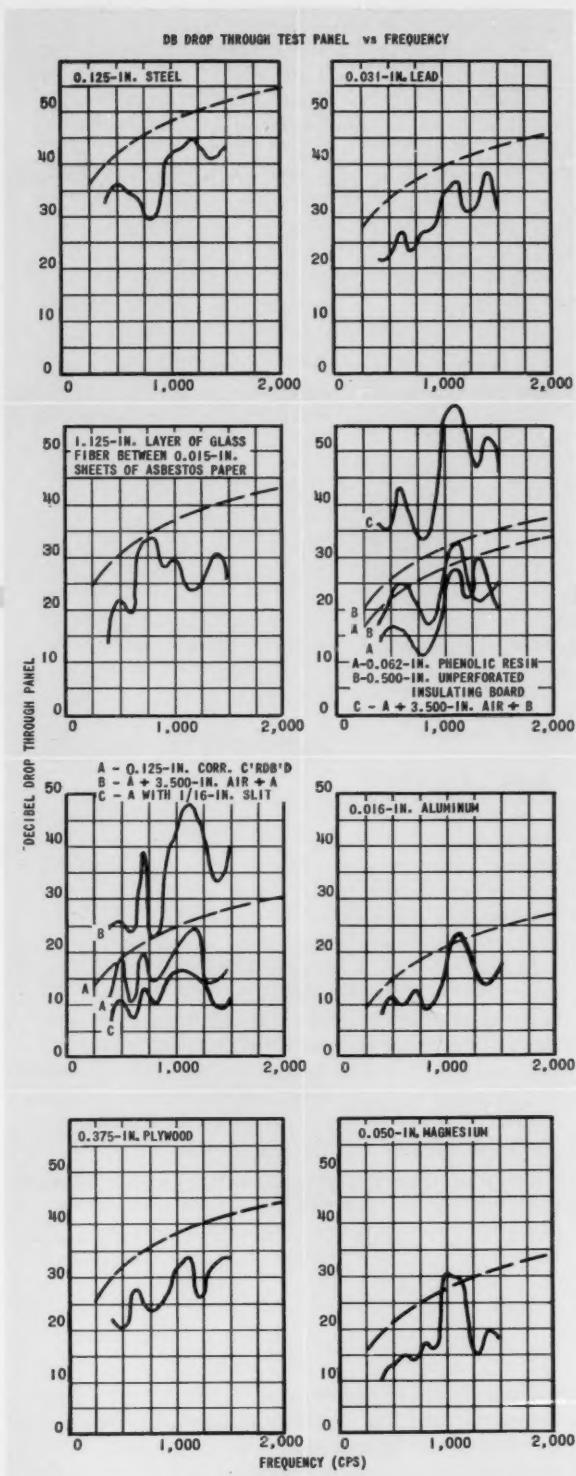
$L$  = wave length of sound

In deriving this formula,<sup>1</sup> some virtually unattainable con-

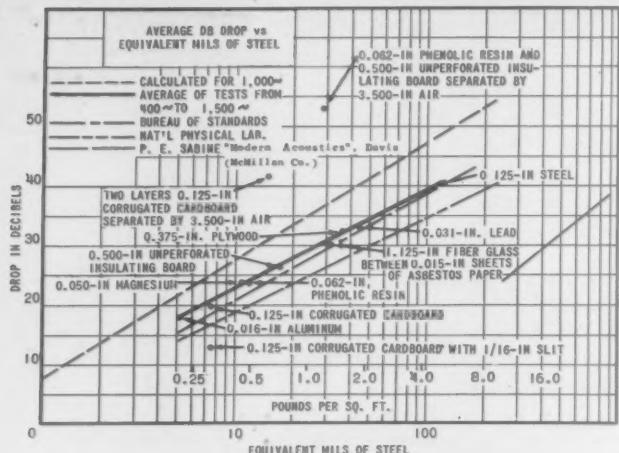
<sup>1</sup> It might be well to point out that tables of natural functions are very convenient in the solution of the above equation. This is accomplished by placing the above equation in the following form:

$$\text{db drop} = 10 \log_{10} \frac{1}{1 - \sin(\tan^{-1} X)}$$

Where:  $X = \pi \times D_1 \times h/D \times L$

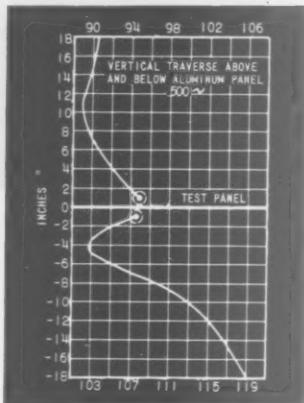


**TEST RESULTS** of materials examined are shown above. All readings were made with the sound-level meter set for a flat, unweighted response. Effects of even a small opening are seen by comparing curves "A" and "C" for corrugated cardboard. (FIG. 2)



**Labeled Points** calculated from test data are shown intersected by the heavy, solid line. Results of other investigators are shown for comparison. The advisability of using a given amount of material in the form of a multiple panel is depicted clearly above. (FIGURE 4)

**STANDING WAVES** on both sides of panel resulted when loud speaker was driven at 500 cycles per second. Correcting this effect called for measuring sound levels as close to the two specimen surfaces as possible. (FIG. 3)



ditions were assumed. For example, the equation holds true only if the panel has no restoring force; actually, the bending moment due to vibration is always present. As a result, a mean curve drawn through the test points parallels the theoretical curve but falls below it.

The variations in the test curves are largely due to two phenomena: the drumhead vibration of the panels, and standing waves. Between 30 and 100 cycles, frequencies well below those tested, the vibration of the thinner panels could be observed by means of sand patterns. In the 400-1,500 cycle test range, however, the vibration was too small to be detected by this means, although it doubtless affected the sound transmission. Similar effects were noticed by the National Physical Laboratory in tests on panels of various common building materials.<sup>2</sup>

Figure 3 shows the results of a vertical traverse above and below the aluminum specimen with the speaker driven at 500 cycles. These particular curves could be extrapolated quite accurately to give the sound level at the two immediate surfaces, but at the highest test frequencies the standing waves varied through such extremes that extrapolation over a distance of one inch was inaccurate.

### Tight panel is essential

The detrimental effect of even a  $\frac{1}{16}$ -inch slit is shown by the sizable difference between curves A and C for the corrugated cardboard panel.

Besides the single panels, two double panels were also tested. The results clearly show that a given amount of sound absor-

<sup>2</sup> Phil. Mag., Vol. 7, Pg. 1050.

ing material, applied as a double panel with an air space between is much more effective than a single layer of the same amount of material.

### Isolated multiple walls are most effective

Initially, the double panels were assembled with a small spacing and with a felt washer under light compression between the two walls. The effectiveness of such a combination was no greater than that of a single panel of double thickness! It was found that the  $\frac{1}{2}$ -inch thick felt loaded at approximately  $\frac{3}{4}$ -lb. per square inch transmitted a surprising amount of vibration. Also, if the faces are not separated by a few inches, the air space between them acts as a film with stiffness calculable from the adiabatic variation of Boyle's Law and communicates increased pressures to the second face with a corresponding increase of transmitted sound. With a free, unloaded felt spacer separating the two walls by  $3\frac{1}{2}$  inches, the double panels gave the expected additive db drop.

### Weight per square foot is controlling factor

The average attenuation from 400 to 1500 cycles is shown in Figure 4 plotted against panel thickness. Rayleigh's formula indicates that the db drop through the panel depends upon the relative masses. For this curve, then, each material was converted to an equivalent steel panel having the same weight per square foot. The average energy represented by the db differences was plotted since this figure is more accurate than the simple arithmetic average of the db drops. The decibels plotted were the average values over the entire frequency range tested.<sup>3</sup> Also indicated on this curve sheet is the theoretical db drop at 1000 cycles and the results of other investigators.

### Applying the results

The tests show a surprising linearity in the relation between the quieting effect of a panel and its weight per square foot. For all the materials tested the decibels' attenuation varies approximately as the .233 power of the ratio of the respective weights per square foot. Thus in choosing material for a wall to stop sound the density or weight per square foot is the controlling factor.

A given amount of material is most effective when applied in the form of a double or multiple panel with air spaces between. Ideally, the two walls should be mechanically separate from each other and from the apparatus being treated. Necessary ventilation should be provided only through acoustically lined ducts or compartments or else be removed as far as possible from the source of the noise within the machine. All joints in the housing should be tight, since even a narrow slit can have a harmful effect on results.

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<sup>3</sup> The average energy represented by a series of db readings is equal to 10 times the logarithm to the base 10 of the average of the antilogarithms  $\frac{1}{10}$  of the separate decibel readings. See *Is it Sound—or Noise?*, L. C. Aicher; A-C Electrical Review, 1st Q '47.

# TRANSIENT OPERATION OF DC Generators



by

DIDIER JOURNEAUX  
Patent Attorney  
Allis-Chalmers Mfg. Co.

*For certain circuit analyses, d-c machines can be assumed to be much simpler than they actually are*

FOR MORE THAN HALF A CENTURY, d-c generators have been of importance mainly as sources of electric power. Their operation under transient conditions has naturally given rise to many problems, the most familiar ones being associated with commutation under variable load. With the more recent application of d-c generators to servo-mechanisms and regulators, closer attention has had to be given to their transient operation under rapidly varying field excitation and the attendant problems which must be solved.

The magnetic field, which is the intangible heart of a d-c generator, causes the field winding to have an inductive character. By delaying the response of the armature voltage to rapid variations in the value of the field voltage, it is a major offender among the causes of sluggishness and of instability of servo-mechanisms and regulators. This article reviews the fundamental aspects of the action of the field winding and the factors which influence the extent of that action.

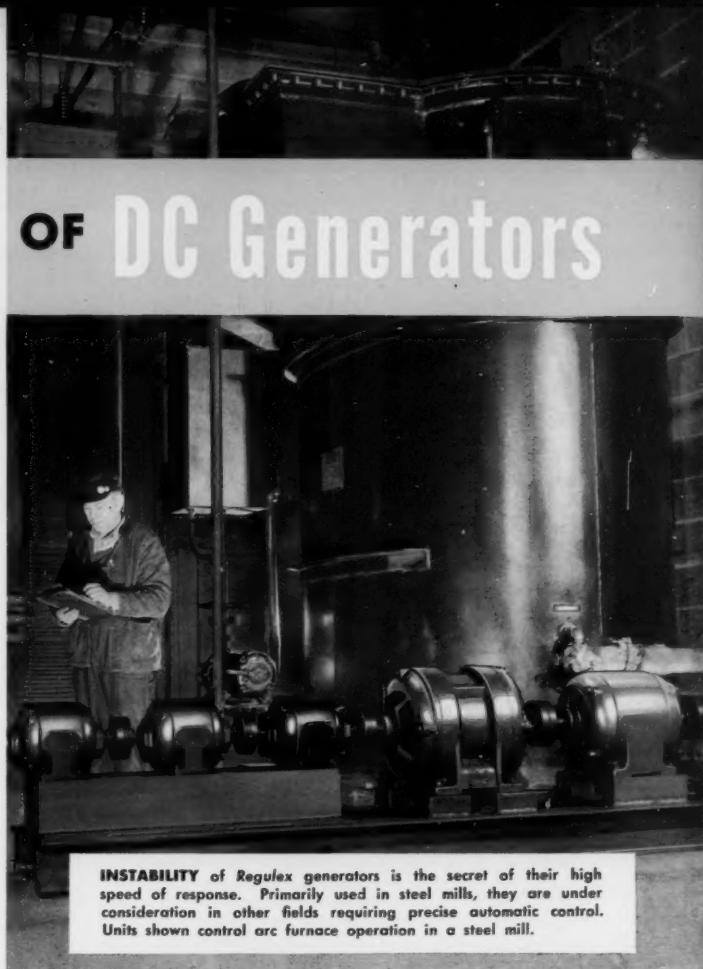
## Consider a single separately-excited field

When a d-c generator has only one field winding, which is separately excited, its transient operation is relatively easy to follow if enough simplifying assumptions are made. Fortunately, computations of sufficient accuracy can often be made by reducing the machine to its bare essentials. This means an armature driven at strictly constant speed, having negligible inductance and capacitance. The armature resistance and its brush drop may be taken into account when computing the armature current.

The magnetic circuit of the machine is optimistically considered to be free of hysteresis, to have a constant permeability, and to be unaffected by armature reaction. Eddy currents will be assumed at first to be nonexistent, although they can be taken into account without much trouble.

The field circuit, including the usual field rheostat, has resistance but will be assumed to have no capacitance and to have only the inductance  $L$  of the field winding. The impedance of the source of excitation will be neglected. The resistance variations caused by temperature changes in the windings and in the field rheostat will be assumed to take place too slowly to require consideration.

In a machine thus idealized, a constant (or variable) field current  $i$  produces in the magnetic circuit a flux which is exactly proportional to it at every instant. This flux in turn



INSTABILITY of Regulex generators is the secret of their high speed of response. Primarily used in steel mills, they are under consideration in other fields requiring precise automatic control. Units shown control arc furnace operation in a steel mill.

causes the armature no-load voltage  $e_a$  to be exactly proportional to the field current at every instant:

$$(1) \quad e_a = ai$$

When the field current has both a constant component and a variable component, the two armature voltage components they produce may be separately computed and added together. This is but an application of the old principle of superposition, whereby effects can be superposed as well as the causes to which they are related by a linear law.

In the generator of Figure 1, if a constant voltage component  $v$  is impressed on the field circuit of resistance  $R$ , it results in a constant field current component of value  $v/R$  and a constant armature voltage component of value  $av/R$ .

## Varying field voltage leads current

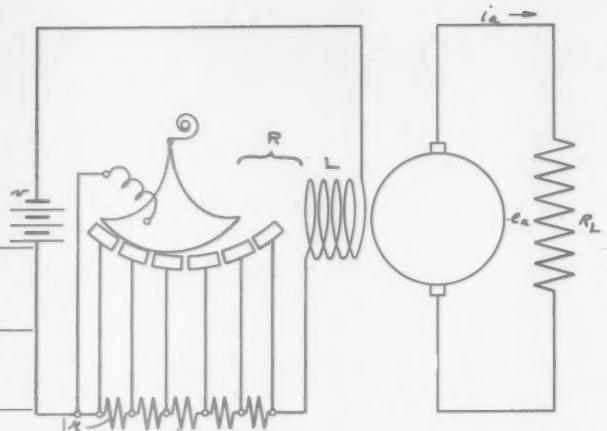
However, if a variable voltage component  $e$  is impressed on the field circuit, the resulting field current component is not directly proportional to it at every instant. The field behaves as any other inductive conductor, and its circuit equation therefore is

$$(2) \quad e = Ri + Lpi$$

where  $p$  is the time derivative operator, also written  $d/dt$ . Solving for  $i$ :

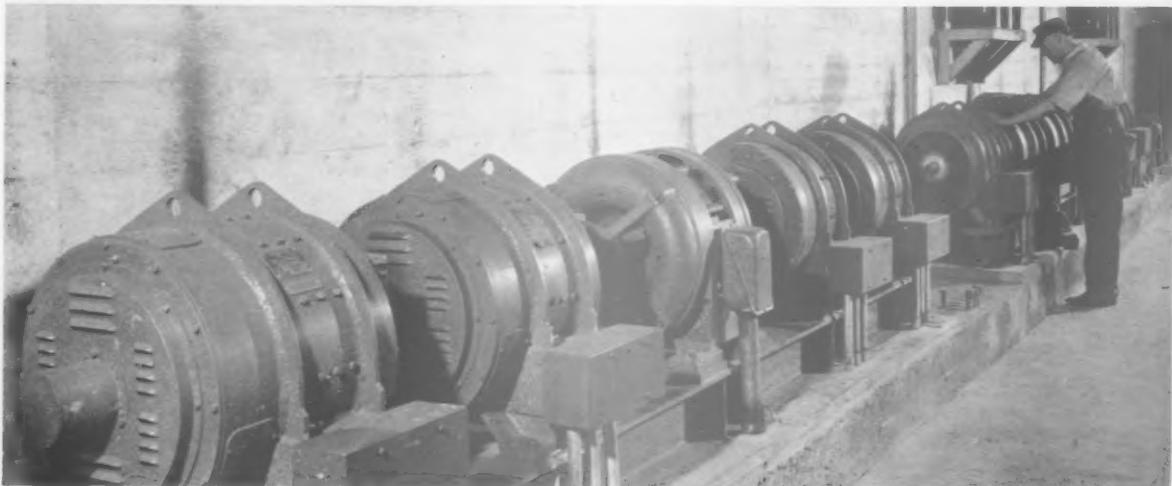
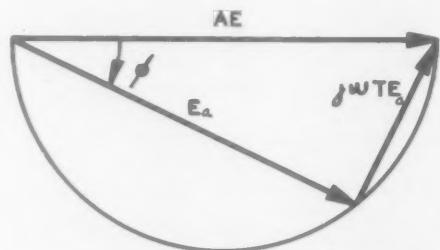
$$(3) \quad i = \frac{e}{R + Lp} = \frac{1}{R} \frac{e}{1 + \frac{L}{R} p}$$

**SINGLE FIELD D-C GENERATOR** diagram shows the field circuit of total resistance  $R$ . A transient is initiated when the regulator removes the short circuit across a section of resistance  $r$ . (FIGURE 1)



**DIMENSIONLESS CURVE** plots the transient components of field current, armature voltage, and load current produced by a step voltage drop in the field circuit of a single field generator. (FIGURE 2)

**VECTOR DIAGRAM** shows the magnitude and phase relations between a sinusoidal voltage impressed on the generator field and the resulting sinusoidal armature voltage. (FIGURE 3)



**NEED FOR PRECISE CONTROL** of speed, tension, voltage, torque, or other variables has brought about the almost complete reliance on rotating

control in the metal rolling industry. This five-machine Regulex set is installed at a leading 48-inch skin pass mill in the midwest.

The value of  $i$  may be substituted into formula (1). In addition, two new coefficients are introduced. One is the time constant  $T$  of the field circuit, which is equal to  $L/R$ , where  $R$ , it should be remembered, is the resistance of the entire field circuit. The other is the voltage gain or amplification  $A$  of the field, which is equal to  $a/R$ . The transient armature voltage component then is of the form

$$(4) \quad e_a = \frac{Ae}{1+Tp}$$

If the armature supplies current to a load circuit without inductance, capacitance or emf, the expression for the transient armature current component is of the same operational form.

The variable field voltage component generally varies at random in actual machines. Its law of variation must be assumed to have one of a few simple forms to permit computing the resulting armature voltage variations.

### Analyzing step field voltage change

The simplest form of variable field voltage component is a step-like voltage change, which may be a rise or a drop. While a rise is more frequently assumed, its mathematical treatment makes it out to be a constant component minus a drop of equal value. The drop, therefore, is somewhat simpler to treat mathematically.

To take an example of practical application, let the excitation circuit of Figure 1, which is operating at a voltage  $v$ , have a total resistance  $R$ . If a section of resistance  $r$  of the rheostat is originally short circuited, the field current has only a constant component of value  $v/(R-r)$ . If the short circuit is suddenly removed, the constant current component takes the value  $v/R$ . In addition, the field circuit carries a transient current component. The latter may be considered to result from the insertion in the circuit of a step voltage drop  $e$  of value  $vr/(R-r)$ . If  $r$  is small relative to  $R$ ,  $e$  is approximately equal to the steady state ohmic drop in the rheostat section inserted in circuit. The solution for the field current transient component in equation (3) is then given as a function of time  $t$  as:

$$(5) \quad i = \frac{e}{R} e^{-\frac{t}{T}}$$

Similarly, the solution for the armature voltage in equation (4) will be

$$(6) \quad e_a = Ae e^{-\frac{t}{T}}$$

If the armature supplies current to a load circuit having only resistance  $R_L$ , the transient armature current component has the value

$$(7) \quad i_a = \frac{Ae}{R_L} e^{-\frac{t}{T}}$$

The generator is thus seen to function as an amplifier in which, given enough time, a field voltage variation  $e$  is amplified to the value  $Ae$ . The field current variations are delayed with respect to the field voltage variations because the store of magnetic energy in the magnetic circuit of the machine is being increased or decreased. However, there is no delay between the variations in the field current, the armature voltage, and the resistive load current. As only one energy storage device is involved, formulas (5), (6) and (7) contain only one time constant. The phenomenon they represent is a single energy transient.

The time variations of the quantities  $i$ ,  $e_a$  and  $i_a$  for any generator may be represented by a single curve if the curve is dimensionless. This characteristic is obtained by taking the ratio of the instantaneous values to their initial values as ordinates and the ratio  $t/T$  as abscissa. The resulting curve is shown in Figure 2.

The initial rate of change of the armature voltage, for example, has the value

$$(8) \quad (pe_a)_0 = -\frac{Ae}{T}$$

If this initial rate of change were maintained, the transient component would reach its final value (zero) at time  $T$ , i.e., when  $t/T$  is equal to unity. Actually, after time  $T$  has elapsed the transient component still has  $100/e$  or 36.7 percent of its initial value. When a time  $4T$  has elapsed, the transient component has decayed to less than two percent of its initial value and may usually be neglected from there on.

Thus when a step voltage variation is impressed on the field winding, the rate of response of the generator is initially relatively high. It gradually decreases as the response is completed.

However, in a relatively unstable system in which the field winding voltage is continually being increased and decreased, the generator will constantly operate at close to its maximum rate of response. A fast rate of response can be obtained by making the amplification large or the field circuit time constant small.

### Analyzing effect of sinusoidal field voltage change

When a d-c generator forms a part of an elaborate servomechanism or regulator, its field may be subjected to voltage changes which have no resemblance to a step variation. It is then useful to know the steady state armature voltage produced by a sinusoidal field voltage, and to plot the variations of the magnitude and of the phase angle  $\phi$  of the armature voltage when the frequency of the field voltage is assumed to vary within an appropriate range.

The vectorial values for  $i$ ,  $e_a$ , and  $i_a$  are given by the familiar formulas of a-c engineering. They may also be obtained by substituting  $j\omega$  for  $p$  in the operational formulas, because the derivative of a sinusoidal time function is equal to the function itself multiplied by  $j\omega$ . Thus, formula (4) gives

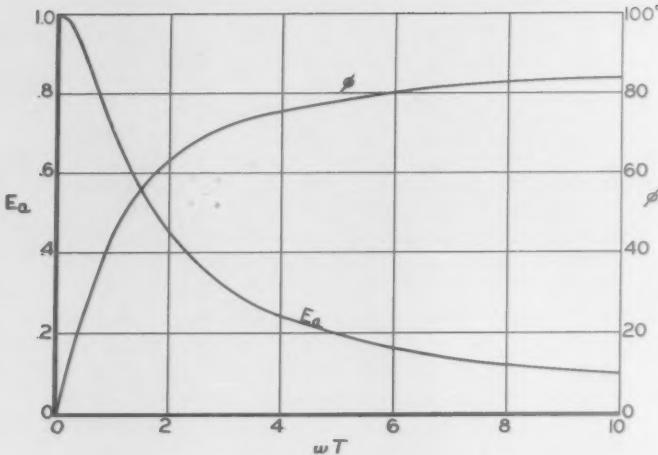
$$(9) \quad E_a = \frac{AE}{1+Tj\omega}$$

$E_a$  is easily obtained in magnitude and in phase by drawing a half circle of diameter  $AE$  and inscribing in the figure a right triangle having its sides proportional to unity and to  $\omega T$ , as shown in Figure 3.  $\phi$  is the familiar phase angle, which is equal to  $\arctan \omega T$ .

Figure 4 gives the variation of  $E_a$  and  $\phi$  in function of  $\omega T$ . The curves are valid for any value of  $T$  because  $\omega T$  is used for abscissas. This renders the abscissas dimensionless,  $\omega$  being but a convenient way of writing  $2\pi$  divided by the period of one alternation. The unit of ordinates for  $E_a$  is  $AE$ , so that the ordinates are also dimensionless. Therefore, the curve is valid for any value of  $A$  and for any other quantities represented by expressions of the same form as  $E_a$ .

By introducing the phase angle  $\phi$  and designating the logarithm of  $\cos \phi$ , which is a negative quantity, as  $-\theta$ , formula (9) can be rewritten:

$$(10) \quad E_a = AE e^{-(\theta + j\phi)}$$



**DIMENSIONLESS CURVES** of the magnitude and phase relations between a sinusoidal field voltage of variable frequency and the resulting sinusoidal armature voltage are shown above. (FIGURE 4)

Formula (10) may not be very serviceable in computations, but is of interest because of its similarity to formula (6). Mathematicians consider that the exponential function is a periodic function having an imaginary period  $2\pi j$ , which accounts for this resemblance.

### Field turns affect amplification

What do the coefficients  $a$ ,  $A$  and  $T$  mean to the design engineer? Coefficient  $a$ , which may be called amplification factor, is the value of the armature voltage per ampere of field current. It is proportional to the number of turns of the field winding in any given unsaturated machine, and can therefore be given a wide range of values by varying the number of field turns.

However, in a well designed d-c generator only the space needed for ventilation is left free around the field coils, to permit reducing the dimensions of the yoke to a minimum. Turns therefore cannot be added to an exciting field winding, and it would not be economical to take off field turns. To change the value of the amplification factor, the available copper volume is divided into the required number of series turns. Of course, the rated field current must vary with the turn cross section.

If it can be assumed that the space factor of the field remains the same when the number of turns is varied, the net total field copper axial cross section is constant. It is equal to the turn cross section times the number of turns.

If the number of turns is changed from  $N$  to  $kN$ , the turn cross section must be made  $k$  times smaller, so that the field circuit resistance increases from  $R$  to  $k^2R$  if there is no field



**USE** of Regulix rotating regulators has extended to a vast number of industries, although the steel industry is still the largest user. This seven machine Regulix set, d-c variable voltage control, and the 1760-kw, 750 rpm synchronous m-g set are installed at a large midwestern skin pass steel mill.

thermostat. The amplification factor becomes  $ka$ , and the amplification  $A/k$ . Actually, the amplification decreases to a greater extent, as the space factor gets lower when a field is replaced by another having a larger number of turns of smaller wire.

### Field copper controls time constant

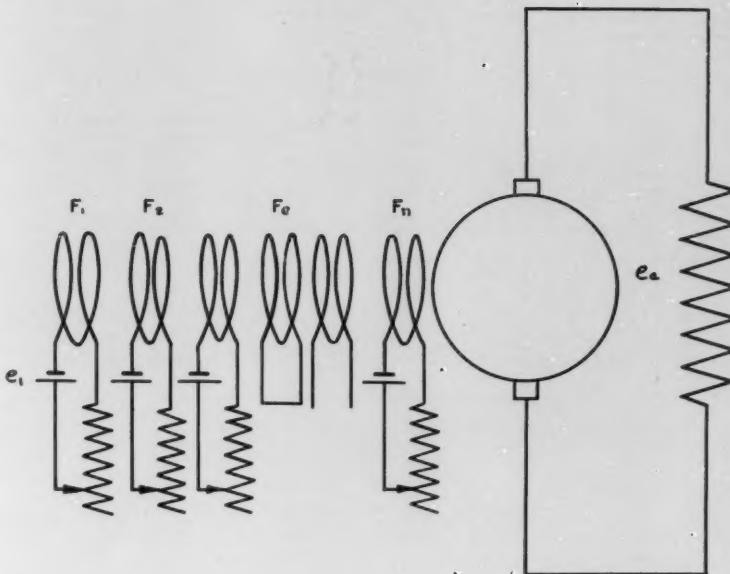
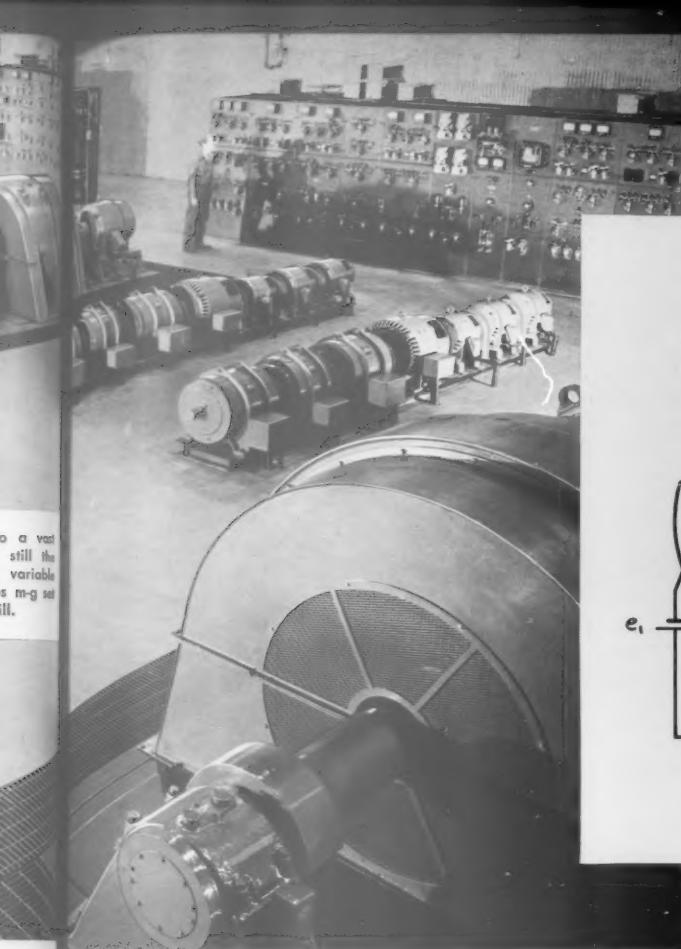
The time constant of the field winding itself is also closely related to the size of the field winding. To find their relation, consider a series of similar generators having different frame sizes. The field inductance of any machine is equal to the number of its flux linkages per ampere, i.e., the number of field turns  $N$  multiplied by the total flux linking with the field winding when the field current is equal to one ampere.

The machines being operated below saturation, the entire reluctance of the magnetic circuit may be assumed to reside in the air gap. The air gap is assumed to have the same width in all machines. The flux per ampere then is proportional to the number of field turns and to the cross sectional area of the air gap. The latter is assumed to be proportional to the square of the mean field turn length  $l$ . This condition is reasonable, as it is met by using the same drawing, at different scales, to determine the transverse cross sections of the field winding and of the air gap in all machines.

By choosing suitable units, the coefficient of proportionality may be made equal to unity, and we may write

$$(11) \quad L = N^2 l^2$$

The resistance of the field winding is proportional to the number  $N$  of field turns and to the length  $l$  of the mean turn, and is inversely proportional to the wire cross section. The latter is equal to the total axial cross section  $c$  of the field



**DIAGRAM SHOWS** the arrangement of elements of a divided field d-c generator. A fictitious short-circuited field simulates the effects of eddy currents in the magnetic circuit of the machine. (FIGURE 5)

copper divided by the number of turns. By again properly choosing the units we may write:

$$(12) \quad R = \frac{NI}{c} = N^2 \frac{1}{c}$$

From (11) and (12) we obtain the time constant of the field winding without field rheostat:

$$(13) \quad T = \frac{L}{R} = lc$$

Under the numerous assumptions we have made, the field time constant of any generator is thus numerically equal to the field copper volume  $lc$ . It is entirely independent of the number of turns of which the field copper is composed. This shows that the field time constant of a machine can be reduced, not by changing the number of field turns, but by reducing the field copper volume. A generally better solution, however, is to reduce the time constant of the field circuit by inserting an appropriate amount of resistance in it. This amount, of course, does depend on the choice of the number of field turns.

If in all machines the magnetic material is best utilized for the same value of the flux density, the field winding should have a constant number of ampere turns. To fully utilize the field copper, its axial cross section  $c$  should be kept constant.

The computation of the field time constant can be extended to generators having different air gap widths. The flux per ampere then varies in inverse proportion to the air gap width. This leads to the conclusion that the field time constant is equal to the field copper volume divided by the air gap width. Of course, the actual value of the field time constant is affected by the fact that a substantial proportion of the field flux does

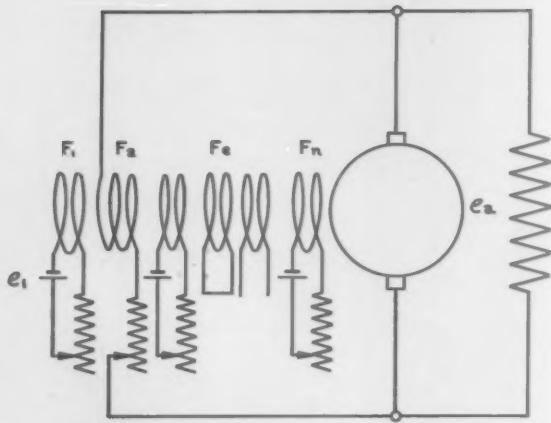
not cross the air gap, and some of it does not even link all the field turns. However, the leakage flux should not disturb the relation between the field time constants of generators of different sizes but similar design.

This single field that we have considered up to this point has been assumed to consist of only one winding disposed on a single magnetic pole. Actually, such a "single" field would consist of a series of windings distributed over two or more poles, but as long as these are connected in series, in parallel, or in series-parallel in a single circuit, their behavior is that of a single coil. But d-c generators used in regulating systems often must be provided with several fields connected in independent circuits.

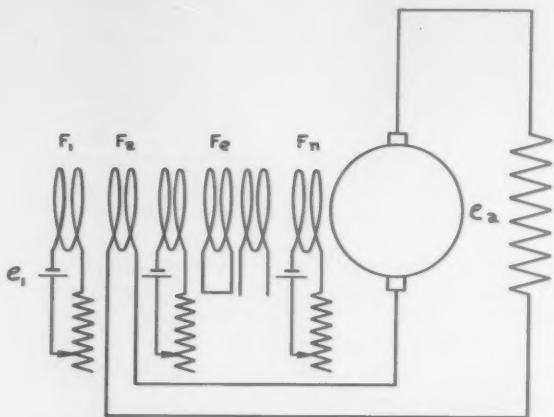
### Consider multiple fields as single field divided

From a practical standpoint, it is generally not feasible to add field windings to an existing machine. To provide several field circuits, an existing single field will therefore have to be divided into  $n$  sections  $F_1, F_2, \dots, F_n$  which are separately energized.

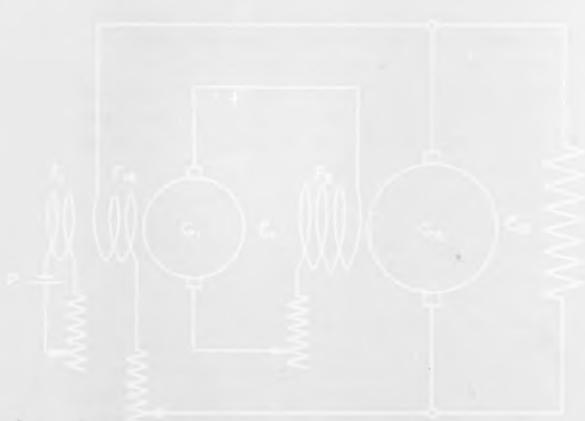
As diagrammatically indicated in Figure 5, the separate field sections may be assumed all disposed together on one pole. The computation of the time constant is the same for a single field and for any winding section. While they may be disposed side by side or have their conductors intermingled, all sections will be assumed to have the same mean turn length and to all link the same flux. In this manner, if the eddy currents in the magnetic circuit can be neglected, the time constants of the single field and of the field sections are proportional to their respective copper volumes. The time constant of the single



**DIVIDED FIELD D-C GENERATOR** with shunt self-energizing field giving positive or negative feedback is diagrammed above. (FIGURE 6)



**DIAGRAM SHOWS** divided field d-c generator with series self-energizing field giving feedback similar to that of shunt field. (FIGURE 7)



**ELEMENTARY FEEDBACK** excitation system connections are shown above. Generator exciter has a separately excited setting field, and a recall field which is excited from the generator. (FIGURE 8)

field then is also necessarily equal to the sum of the time constants of the field sections.

The field sections are all endowed with resistance and self inductance. Taken two by two, they also have mutual inductance due to their common magnetic flux. The major part of this flux passes through the air gap into the armature to produce the armature voltage. The values of the amplifications  $A_1, A_2, \dots, A_n$  obtained by means of the different sections take into account the stray flux which passes through all the field sections but not through the armature.

As the field sections link the same flux, their coupling is unity and their mutual inductances are maximum. There is no leakage flux in the sense of non-common flux of the different windings of a transformer, and the different sections have zero leakage reactance. This assumption, which is reasonably warranted, permits a simple mathematical treatment of machines which would otherwise be quite unmanageable. A single flux common to all field sections causes the magnetic circuit of the machine to behave as a single energy storage device. The circuit beyond the field winding terminals is assumed to contain only sources and resistive elements, so that the machine becomes involved only in single energy transients.

#### Handling variable section voltage problem

If variable voltage components  $e_1, e_2, \dots$  etc. are impressed on two or more field sections, the armature voltage is given by the relation

$$(14) \quad e_a = \frac{\sum A_m e_m}{1 + \sum T_m p}$$

which is derived in the appendix. The machine behaves as if it had a single field on which the variable voltage components, weighted to give them the proper amplification, are impressed in series. The machine time constant  $\sum T_m$  is a single time constant equal to the sum of the time constants of all the circuits in which individual field sections are connected.

If the field winding connections have appreciable resistance, the time constants of the field circuits are lower than the time constants of the field sections alone. The machine time constant then is certainly lower than the time constant of the single field.

If the field winding sections are connected to sources of negligible impedance through connections whose impedance is likewise negligible, the sum of the time constants of the excitation circuits is the sum of the time constants of the field sections connected therein. If all the field sections are connected, this sum is equal to the time constant  $T$  of the single field. In this special case,

$$(15) \quad e_a = \frac{\sum A_m e_m}{1 + T_p}$$

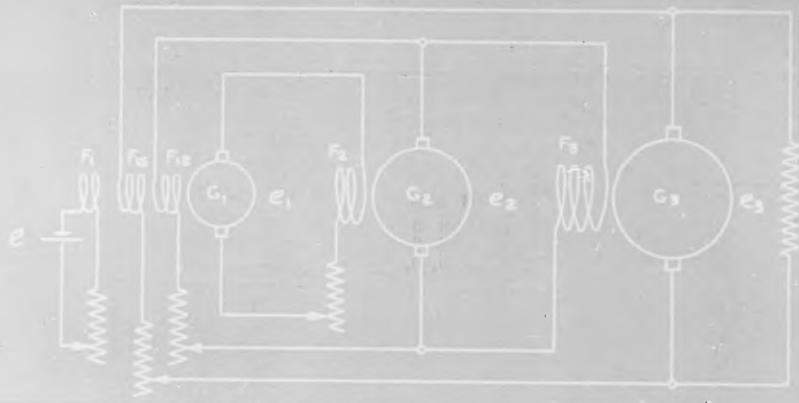
If only one field section  $F_1$  is impressed with a variable voltage component, the transient current components in the other sections are only the components induced by transformer action in those sections which are in closed circuits. As far as the computation of transient components is concerned, those sections are short circuited on their field rheostats even if they actually are connected to sources of constant voltage. The transient armature voltage component then is

$$(16) \quad e_a = \frac{A_1 e_1}{1 + \sum T_m p}$$

If  $e_1$  is a step voltage drop, the solution for equation (16) is

$$(17) \quad e_a = A_1 e_1 \cdot \frac{-t}{\sum T_m}$$

**CONNECTIONS** of feedback excitation system for a generator with a main exciter and pilot exciter. The latter is provided with setting, balancing, and recall fields. (FIGURE 9)



The initial rate of change of the armature voltage has the value

$$(18) \quad (pe_a)_0 = \frac{-A_1 e_1}{\Sigma T_m}$$

Relative to a single field machine, the rate of response is reduced in the ratio of the field section amplification  $A_1$  to the amplification  $A$  of the entire field.

### Eddy currents act as short circuited field

So far the magnetic circuit of the machine has been assumed to be free of eddy currents. Actually, magnetic flux variations always result in eddy currents in the cores and yokes of the field. Of course eddy currents are largest in machines with solid magnetic elements, but their effect cannot be neglected even in machines with laminated cores and yokes. The nature of this effect results from the fact that each line of force of the magnetic flux links with some eddy currents in the manner that the entire magnetic flux links with the field windings. The result is roughly as if a single eddy current were linking with the entire magnetic flux.

From a practical standpoint, it is therefore satisfactory to assume that the eddy currents are replaced by an additional short circuited field section linking the entire magnetic flux. With this assumption, there is no such thing as a single field machine. There is always the fictitious eddy current field section besides the energized field. The sum of their two time constants is approximately equal to the apparent time constant obtained by substituting in formula (18) the experimental values of rate of response and of initial armature voltage. In computing the time constants of the sections of a single field proportionally to their copper volumes, the value of the time constant of the single field should not include the effect of eddy currents. The eddy current time constant therefore should be subtracted from the experimental time constant of the single field and should be allocated to the fictitious field section.

### Handling the self energizing field

Let the generator of Figure 5 be provided with  $n$  field sections, of which only section  $F_1$  is impressed with a variable voltage component  $e_1$  and section  $F_2$ , for example, is the fictitious short circuited eddy current section. Furthermore, let field section  $F_2$  be reconnected either as a shunt field, Figure 6, or as a series field, Figure 7. As a result of this reconnection,  $e_2$  is the armature voltage  $e_a$  and formula (14) gives

$$(19) \quad e_a = \frac{A_1 e_1 + A_2 e_n}{1 + \Sigma T_m p} = \frac{A_1}{1 - A_2} \frac{e_1}{1 + \frac{\Sigma T_m}{1 - A_2} p}$$

If the magnetomotive forces of fields  $F_1$  and  $F_2$  are additive in the magnetic circuit of the machine, self excitation field  $F_2$  has a positive feedback effect. In this way, the amplification given by field  $F_1$  is increased from  $A_1$  to  $A_1/(1 - A_2)$ .

If  $A_2$  is less than unity,  $R_2$  is larger than the amplification factor of field  $F_2$  and the resistance line of field  $F_2$  is steeper than the saturation curve of the machine. Self excitation alone is insufficient to induce any voltage in the armature, but it reinforces the effects of the other fields. The time constant of the machine is increased from  $\Sigma T_m$  to  $\Sigma T_m/(1 - A_2)$ , where  $\Sigma T_m$  is still the sum of the time constants of all the excitation circuits, including the self excitation circuit. The initial rate of change of the armature voltage in response to a step voltage drop  $e_1$  remains  $-A_1 e_1 / \Sigma T_m$ . The shunt or series field, while increasing the amplification of the control field, thus leaves the initial speed of response of the machine unimpaired. Only the completion of the response is delayed.

If  $A_2$  is increased to unity, the resistance line of field  $F_2$  is brought into coincidence with the saturation curve given by field  $F_2$ . This means that the current in field  $F_1$  only needs to overcome the hysteresis of the magnetic circuit to start its magnetization toward zero or toward saturation. When the current in field  $F_1$  vanishes, self-excitation alone maintains whatever magnetization the magnetic circuit had reached and maintains the armature voltage at the corresponding value.

If  $A_2$  is raised above unity, the machine excites itself to a voltage determined by the saturation of its magnetic circuit. As self-excitation is increased, the machine gradually becomes less sensitive to separate excitation.

By reversing the connections between field  $F_2$  and the armature, the field is given a negative feedback effect and reduces the amplification to  $A_1/(1 + A_2)$ . The value of  $A_2$  then must be less than unity, as otherwise field  $F_2$  would nullify the action of field  $F_1$ . The amplification accordingly remains above  $A_1/2$ . The time constant of the machine becomes equal to  $\Sigma T_m/(1 + A_2)$ . If  $T_1$  constitutes a large part of  $T_m$ , the time constant of the machine thus may be reduced below  $T_1$ . Again the initial rate of change of the armature voltage remains  $-A_1 e_1 / \Sigma T_m$ .

If variable voltage components are impressed at the same time on different fields, the resulting armature voltage may be computed by formula (14).

### Feedback systems are special problem

Multiple field generators find wide application in feedback excitation systems. In its simplest form shown in Figure 8, such a system comprises a generator  $G_2$  of amplification  $A_2$  having a field  $F_2$  connected to an exciter  $G_1$ . The latter has a

setting field  $F_1$  and a recall field  $F_{12}$  giving amplifications  $A_1$  and  $A_{12}$  respectively. Field  $F_{12}$  bucks field  $F_1$  and therefore has a negative feedback effect. In applying formula (16) to single field generator  $G_2$ , let  $T_2$  represent the sum of the time constants of the circuit of field  $F_2$  and of the eddy currents in  $G_2$ :

$$(20) \quad e_2 = \frac{A_2 e_1}{1 + T_2 p}$$

Likewise, in applying formula (14) to exciter  $G_1$ , let  $T_1$  represent the sum of the time constants of the circuits of fields  $F_1$  and  $F_{12}$  and of the eddy currents in  $G_1$ :

$$(21) \quad e_1 = \frac{A_1 e - A_{12} e_2}{1 + T_1 p}$$

Equating the values of  $e_1$  in formulas (20) and (21) gives

$$(22) \quad e_2 = \frac{A_1 A_2 e}{(1 + T_1 p)(1 + T_2 p) + A_2 A_{12}}$$

In the absence of field  $F_{12}$ , the steady state value of  $e_2$  would be  $A_1 A_2 e$ . Because of field  $F_{12}$ , this value is reduced to  $A_1 A_2 e / (1 + A_2 A_{12})$ , which usually is close to  $A_1 e / A_{12}$ . Depending upon the relative values of  $T_1$ ,  $T_2$  and  $A_1 A_{12}$ , the armature voltage component resulting from a step voltage component in field  $F_1$  may take different forms. As indicated by the form of formula (22), the response may be overdamped, critically damped or damped oscillatory.

A more elaborate feedback system is shown in Figure 9. In this system generator  $G_1$  is a pilot exciter and generator  $G_2$  a main exciter. Field  $F_1$  of generator  $G_1$  is bucked by recall field  $F_{13}$  and by a balancing field  $F_{13}$  energized from the main generator  $G_3$ . Formula (20) again applies to exciter  $G_2$ . A similar formula applies to the main generator:

$$(23) \quad e_3 = \frac{A_3 e_2}{1 + T_3 p}$$

Taking into account field  $F_{13}$ , formula (14) gives for the pilot exciter:

$$(24) \quad e_1 = \frac{A_1 e - A_{12} e_2 - A_{13} e_3}{1 + T_1 p}$$

Formulas (20), (23) and (24) give

$$(25) \quad e_3 = \frac{A_1 A_2 A_3 e}{(1 + T_1 p)(1 + T_2 p)(1 + T_3 p) + (1 + T_3 p) A_2 A_{12} + A_2 A_3 A_{13}}$$

In this system also the response to a step voltage component may be overdamped, critically damped or damped oscillatory. In each case, however, the law of variation of  $e_3$  involves an additional exponential term. Depending upon the characteristics of the machines, the system may also enter into sustained oscillations; this can be cured by the provision of suitable damping means.

## APPENDIX

Field sections  $F_1$  to  $F_n$  of Figure 5 having  $N_1$  to  $N_n$  turns and amplification factors  $a_1$  to  $a_n$  are connected in circuits of resistances  $R_1$  to  $R_n$ . They have self inductances  $L_1$  to  $L_n$  and mutual inductances  $M_{1,2}$  to  $M_{n-1,n}$ . The field currents  $i_1$  to  $i_n$  are related to the field voltages  $e_1$  to  $e_n$  by a system of  $n$  linear equations:

$$(26) \quad \begin{aligned} e_1 &= R_1 i_1 + L_1 p i_1 + \sum M_{1,m} p i_m \\ e_2 &= R_2 i_2 + L_2 p i_2 + \sum M_{2,m} p i_m \\ &\text{etc. to} \\ e_n &= R_n i_n + L_n p i_n + \sum M_{n,m} p i_m \end{aligned}$$

In each equation the subscript  $m$  assumes all values from 1 to  $n$  except the other subscript.

The resulting armature voltage is ( $m$  having all values from 1 to  $n$ ),

$$(27) \quad e_a = \sum a_m i_m$$

Because of the absence of leakage reactances, the relations between the self and mutual inductances of the sections taken two by two are of the form

$$(28) \quad M_{p,q} = \sqrt{L_p L_q}$$

As the sections all link the same flux, their self-inductances are proportional to the square of their numbers of turns. Hence

$$(29) \quad \frac{L_p}{(N_p)^2} = \frac{L_q}{(N_q)^2} = \sqrt{\frac{L_p L_q}{(N_p)^2 (N_q)^2}} = \frac{M_{p,q}}{N_p N_q}$$

Also because all sections link the same flux:

$$(30) \quad \frac{a_q}{N_q} = \frac{a_p}{N_p}$$

Multiplying (29) by (30) gives

$$(31) \quad L_p \frac{a_q}{a_p} = M_{p,q}$$

Equations (26) may therefore be rewritten

$$(32) \quad \begin{aligned} e_1 &= R_1 i_1 + L_1 p \sum \frac{a_m i_m}{a_1} \\ e_2 &= R_2 i_2 + L_2 p \sum \frac{a_m i_m}{a_2} \\ &\text{etc. to:} \\ e_n &= R_n i_n + L_n p \sum \frac{a_m i_m}{a_n} \end{aligned}$$

where  $m$  is given all values from 1 to  $n$ . From (27) and (32)

$$(33) \quad \begin{aligned} e_1 &= R_1 i_1 + L_1 p \frac{e_a}{a_1} \\ e_2 &= R_2 i_2 + L_2 p \frac{e_a}{a_2} \\ &\text{etc. to:} \\ e_n &= R_n i_n + L_n p \frac{e_a}{a_n} \end{aligned}$$

Solving (33) for the currents

$$(34) \quad \begin{aligned} i_1 &= \frac{e_1}{R_1} - T_1 p \frac{e_a}{a_1} \\ i_2 &= \frac{e_2}{R_2} - T_2 p \frac{e_a}{a_2} \\ &\text{etc. to:} \\ i_n &= \frac{e_n}{R_n} - T_n p \frac{e_a}{a_n} \end{aligned}$$

Substituting (34) in (27)

$$(35) \quad e_a = \sum A_m e_m - \sum T_m p e_a$$

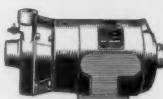
Solving (35) for  $e_a$ :

$$(14) \quad e_a = \frac{\sum A_m e_m}{1 + \sum T_m p}$$

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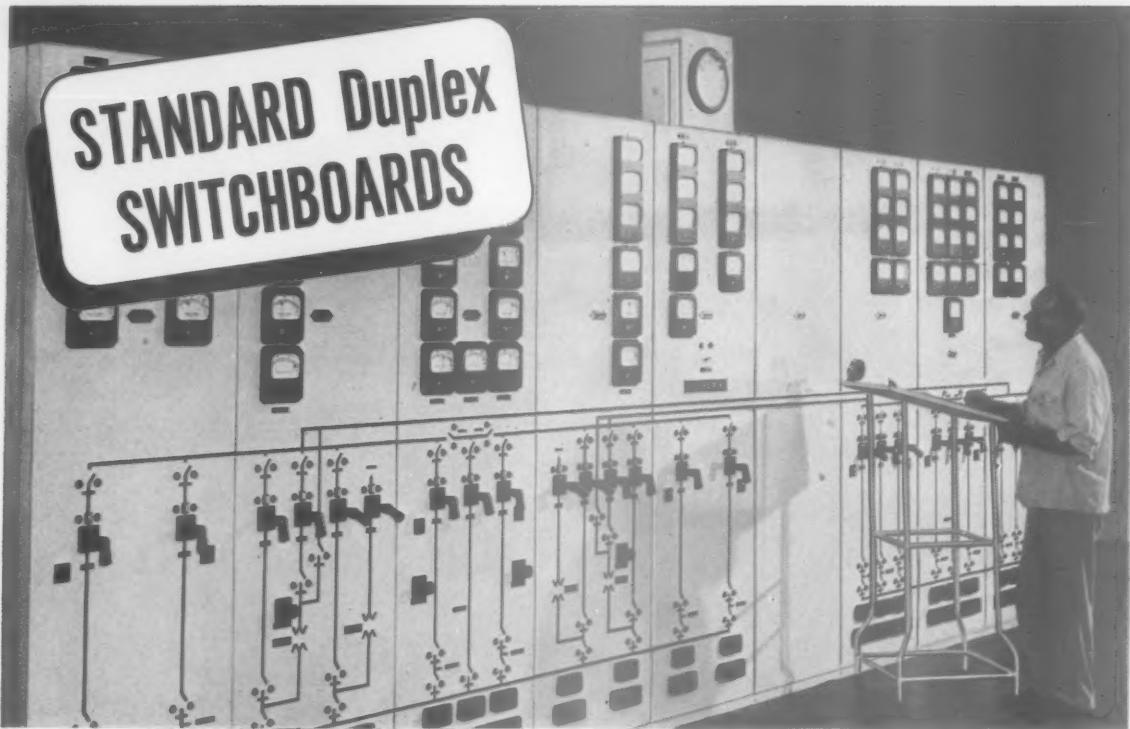
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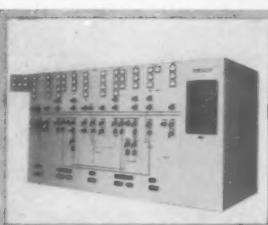
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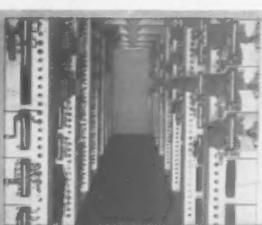
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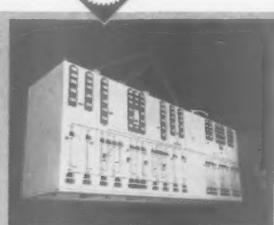
Standardized. This Duplex board, remotely controls 69 kv outdoor circuit breakers, has standard instruments and controls. Mimir bus identifies circuits.



Convenient. Large access doors at each end and well-illuminated interior permit easy inspection. Wiring troughs protect wiring. Terminal blocks are easy to get at.



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